Channel estimation for block-fading frequency-selective Rayleigh MIMO channels: performance limits

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Abstract— **In multiple antenna systems, channel estimation is a critical issue due to the large number of parameters to be estimated. For system analysis and design, it is of theoretical and practical relevance to derive a lower bound on the correlation matrix of the channel estimation error. In this paper, we derive a lower bound that can be asymptotically attained for a large number of blocks in a block-fading channel.**

For two different frequency-selective channel models, the analytic bound is investigated for practical system settings through numerical results in order to show the impact of Doppler spectrum, spatial correlation, number of multipaths, number of transmit/receive antenna on channel estimation performance.

I. INTRODUCTION

For system analysis and desing of multi-antenna links (MIMO systems), the effects of channel estimation errors have to be appropriately taken into account. In a recent publication [2], a lower bound on the channel estimation error has been derived for a MIMO time-slotted system in which each time slot contains a training period and (one or more) payload section(s). The propagation channel was assumed to be frequency-selective and constant within each time-slot (block fading [3]). The bound was obtained in [2] by computing the asymptotic performance of a parametric estimator that fully exploits the structure of the multipath channel and was proved to reduce to known results in simplified settings. Moreover, this derivation (here sketched in Sec. II) has been shown to provide the same analytic bound that would be obtained by computing the (hybrid) Cramer Rao Bound for the problem at hand [4].

The lower bound holds for two different multipath MIMO channel models (and models obtained by combination of these) corresponding to different assumptions about the geometry of antenna arrays and scatterers

1) *Beamforming model* [5]: the elements of both transmitting and receiving antenna arrays are co-located and the scatterers can be considered as point sources. Each path of the multipath channel is characterized by a direction of departure (DOD) and of arrival (DOA), a delay and a complex amplitude (fading). The latter is in turn modelled as a temporally correlated Gaussian stationary process. As a general rule, this model appears to be well suited for outdoor channels [6] and has been used in a SIMO context in, e.g., [7];
2) *Diversity model* [3]: the elemens of both the transmitting and receiving antenna arrays are not co-located and/or the different scatterers have to be modelled as distributed sources. These assumptions are generally appropriate for an indoor scenario [8]. For each delay of the multipath, the channel gains between different transmitting and receiving antennas can be modeled as spatially and temporally correlated jointly Gaussian random variable with zero mean (Rayleigh fading). This model has been used (with some simplifications) by [9] in the context of MIMO-OFDM transmission.

In this paper, the analytic results of [2] are investigated for practical systems through numerical results in order to show the impact of Doppler spectrum, spatial correlation, number of multipaths, number of transmit/receive antenna on channel estimation performance.

The paper is organized as follows: Sec. II presents the signal and channel models under which the lower bound on channel estimation error is briefly derived. For a thorough presentation of the material covered by this Section, the reader is referred to [2]. The impact of system parameters is then investigated through numerical results in Sec. III. In particular, the effect of different Doppler spectra is investigated in Sec. III-A, the performance loss of suboptimal (and computatially less intensive) approaches to channel estimation is studied in Sec. III-B and finally Sec. III-C discusses the impact of spatial correlation of fading amplitudes.

Notation: lowercase (uppercase) bold denotes column vector (matrices), $(\cdot)^T$ is the matrix transpose, $(\cdot)^H$ is the Hermitian transposition,$\otimes$ is the Kronecker matrix product, $\mathbf{v} = \mathrm{vec}\{\mathbf{V}\}$ is the stacking operator and the following property is extensively used: $\mathrm{vec}\{\mathbf{ABC}\} = (\mathbf{C}^T \otimes \mathbf{A})\,\mathrm{vec}\{\mathbf{B}\}$. $\mathbf{I}_P$ is the $P \times P$ identity matrix, $\mathbf{R}^{1/2}$ is a factorization of $\mathbf{R}$ such that $\mathbf{R}^{H/2}\mathbf{R}^{1/2} = \mathbf{R}$.

II. LOWER BOUND ON THE CHANNEL ESTIMATION MSE

As detailed in [2], the time-varying multipath channel has some characteristics that are stationary (or varying over a long-term) and other features that are fast-varying (e.g., fading amplitudes). A simple example can illustrate the different varying rates: while the fading amplitudes can vary completely when either end of the communication link moves as little as $\lambda/4$ (λ is the carrier wavelength), the angles (DOA's and DOD's) remain constant with changes in position of several wavelengths (i.e., 10 to 1000λ). The entries of the frequency-selective MIMO channel matrix within the kth slot can be arranged into a vector

$$\mathbf{h}_k = \mathbf{U}\mathbf{d}_k \tag{1}$$

so that the slowly varying term, represented by the matrix $\mathbf{U}$, and the fast varying fading vector $\mathbf{d}_k$ can be decoupled. We consider an estimator that: a) is able to consistently estimate the long term features of the channel $\mathbf{U}$ so that for a large number of bursts K (ideally $K \to \infty$) these can be assumed to be acquired with any accuracy; b) performs optimum (minimum mean square error, MMSE) tracking of the variations of the fast-varying features $\mathbf{d}_k$. Notice that many known estimators proposed in the literature under simplified settings have (at least one of) the aforementioned properties [7] [10]. By deriving the asymptotic ($K \to \infty$) error correlation matrix

$$\mathbf{Q}_{\hat{\mathbf{h}}} = E[(\hat{\mathbf{h}}_k - \mathbf{h}_k)(\hat{\mathbf{h}}_k - \mathbf{h}_k)^H] \tag{2}$$

or the corresponding mean square error (MSE)

$$MSE_{\hat{\mathbf{h}}} = E[(\hat{\mathbf{h}}_k - \mathbf{h}_k)^H(\hat{\mathbf{h}}_k - \mathbf{h}_k)] = \mathrm{tr}\{\mathbf{Q}_{\hat{\mathbf{h}}}\}. \tag{3}$$

of the estimate for this method, we set a lower bound on the channel estimation error over the considered channel models (as it can be shown analytically through computation of the hybrid Cramer Rao Bound [4]).

The lower bound, whose derivation is briefly recalled in the following Sections, reads

$$\mathbf{Q}_{\hat{\mathbf{h}}} = \mathbf{U}\int_{-\pi}^{\pi} S_\varphi(\omega)(\mathbf{R}_d^{-1} + S_\varphi(\omega)\mathbf{R}_w)^{-1}\frac{d\omega}{2\pi}\mathbf{U}^H. \tag{4}$$

where

- $S_\varphi(\omega)$ is the Doppler spectrum
- $\mathbf{R}_d = E[\mathbf{d}_k\mathbf{d}_k^H]$ is the correlation matrix of the fast varying fading vector (see Sec. II-B)
- $\mathbf{R}_w = \mathbf{U}^H(\mathbf{R}_x \otimes \mathbf{R}_n^{-1})\mathbf{U}$, where
  - $\mathbf{R}_x$ is the correlation matrix of the training sequences (see Sec. II-A) and
  - $\mathbf{Q}$ is the spatial covariance matrix of the additive Gaussian noise (see Sec. II-A).

A. Signal model

A MIMO link with M receiving and N transmitting antennas over a frequency-selective fading channel constitutes the scenario under which (4) is derived. The discrete-time baseband model of the received signal at symbol rate $1/T$ within the training period can be written as

$$\mathbf{y}_k[\ell] = \sum_{i=0}^{W-1} \mathbf{H}_k[i]\mathbf{x}[\ell - i] + \mathbf{n}_k[\ell], \tag{5}$$

where $\mathbf{y}_k[\ell]$ is the $M \times 1$ vector received over the M receiving antennas, W is the temporal support of the multipath channel, $\mathbf{x}[\ell]$ is the $N \times 1$ vector containing the ℓth training symbols for the N transmitting antennas, the W matrices $M \times N$ $\{\mathbf{H}_k[i]\}_{i=0}^{W-1}$ constitute the W MIMO taps of the multipath channel, the additive Gaussian noise $\mathbf{n}_k[\ell]$ is assumed temporally uncorrelated but spatially correlated with covariance matrix $\mathbf{Q}$: $E[\mathbf{n}_k[\ell]\mathbf{n}_k[\ell - i]^H] = \mathbf{Q}\delta[i]$. Collecting the received L samples of the training sequences into the $M \times L$ matrix $\mathbf{Y}_k = [\mathbf{y}_k[0], ..., \mathbf{y}_k[L-1]]$ we get

$$\mathbf{Y}_k = \mathbf{H}_k\mathbf{X} + \mathbf{N}_k, \tag{6}$$

where $\mathbf{H}_k = [\mathbf{H}_k[0], \mathbf{H}_k[1], ..., \mathbf{H}_k[W-1]]$ is the $M \times NW$ MIMO-FIR channel matrix, $\mathbf{X}$ is the $NW \times L$ convolution matrix obtained from the N training sequences so that the ith column of $\mathbf{X}$ is $[\mathbf{x}[i]^T, \mathbf{x}[i-1]^T, ..., \mathbf{x}[i-W+1]^T]^T$, noise $\mathbf{N}_k$ has the same structure as $\mathbf{Y}_k$ and $1/L \cdot E[\mathbf{N}_k\mathbf{N}_k^H] = \mathbf{Q}$. The spatial (among the transmitting antennas) and temporal correlation of the training sequences is given by the matrix $\mathbf{R}_x = \mathbf{X}^*\mathbf{X}^T$ (see next Section).

In MIMO-OFDM systems, the received signal has the same structure as in (6). For pilot-based channel estimation, K subcarriers are used to carry pilot symbols. Therefore, for channel estimation purposes the received signal can be collected in a $M \times K$ matrix $\mathbf{Y}_k$ that contains the frequency-domain samples received over the pilot subcarriers. Matrix $\mathbf{H}_k$ is the $M \times NW$ MIMO-FIR (time-domain) channel matrix defined as in (6). The $NW \times K$ matrix $\mathbf{X}$ in this case is no longer a convolution matrix but is the product of a matrix that perform MN Discrete Fourier Transforms on the time-domain channels in $\mathbf{H}_k$ and a block-diagonal matrix that gathers the pilot symbols transmitted by the N transmitting antennas. Notice that the design of training sequences (or pilot symbols) can be optimally performed so as to decouple the (non-parametric) channel estimation for different transmitting antennas [12] [13]. We emphasize that even in this case, a parametric estimator can benefit from a joint estimate of the channels for all the transmitting antennas (see Sec. III-B). Since the signal model for single-carrier and multi-carrier (MIMO-OFDM) is the same (6), the discussion in the following can be easily applied to the latter as well.

For channel estimation it is convenient to arrange the L columns of the matrix $\mathbf{Y}_k$ into a $ML \times 1$ vector $\mathbf{y}_k = \mathrm{vec}\{\mathbf{Y}_k\}$ so that the model (6) becomes

$$\mathbf{y}_k = (\mathbf{X}^T \otimes \mathbf{I}_M)\mathbf{h}_k + \mathbf{n}_k = \bar{\mathbf{X}}\mathbf{h}_k + \mathbf{n}_k, \tag{7}$$

the columns of $\mathbf{H}_k$ are similarly stacked into a $MNW \times 1$ vector $\mathbf{h}_k = \mathrm{vec}\{\mathbf{H}_k\}$.

B. Channel model

The channel vector $\mathbf{h}_k$ in (7) can be parametrized as the combination of d paths, where each path has different features according to the propagation environment (beamforming or diversity). It can be shown that $\mathbf{h}_k$ can be written in terms of the channel parameters as [2]

$$\mathbf{h}_k = \mathbf{T}\boldsymbol{\beta}_k. \tag{8}$$

Matrix $\mathbf{T}$ is stationary across a large number of bursts (say K) as it depends on the DOD's, DOA's, delays and power profile of fading. In order to reduce the physical model (8) to the algebraic structure of the channel (1), we consider the following. Different paths having the similar propagation parameters do not always contribute to the matrix $\mathbf{T}$ with linearly independent columns (see [11] for a discussion in the context of SIMO systems). In fact, when the separation between delays (and/or angles) is below the temporal (and spatial resolution), the matrix $\mathbf{T}$ is rank-deficient. Therefore, matrix $\mathbf{T}$ is not identifiable. In order to write the model (8) in terms of an equivalent minimum and identifiable parametrization it is thus necessary to introduce a full rank matrix $\mathbf{U}$ such that $\text{span}\{\mathbf{T}\} = \text{span}\{\mathbf{U}\}$. A convenient choice is to select $\mathbf{U}$ as an orthonormal matrix, i.e., in terms of the singular value decomposition of $\mathbf{T}$, $\mathbf{T} = \mathbf{U}\mathbf{\Lambda}^{1/2}\mathbf{V}^H$, obtaining (1) with $\mathbf{d}_k = \mathbf{\Lambda}^{1/2}\mathbf{V}^H\boldsymbol{\beta}_k$.

We define for the ith path the following quantitites: $\mathbf{a}_T(\alpha_i^{(T)})$ the $N \times 1$ transmitting array response (corresponding to DOD $\alpha_i^{(T)}$), $\mathbf{a}_R(\alpha_i^{(R)})$ the $M \times 1$ receiving array response (corresponding to DOA $\alpha_i^{(R)}$), $\mathbf{g}(\tau_i)$ the $W \times 1$ transmitted waveform delayed by τ_i, Ω_i the received power. Then, collecting the power profile $\{\Omega_i\}_{i=1}^d$ in a $d \times d$ diagonal matrix $\mathbf{\Omega}_k$, we can specialize model (8) (and as a consequence (1)) as [2]

- Beamforming model:

$$\mathbf{T} = [\Omega_1^{1/2}\mathbf{g}(\tau_1) \otimes \mathbf{a}_T(\alpha_1^{(T)}) \otimes \mathbf{a}_R(\alpha_1^{(R)}), \cdots, \Omega_d^{1/2}\mathbf{g}(\tau_d) \otimes \mathbf{a}_T(\alpha_d^{(T)}) \otimes \mathbf{a}_R(\alpha_d^{(R)})] \quad (9)$$

with space-time correlation of the fading amplitudes $\boldsymbol{\beta}_k$ (φ_n is the temporal correlation function, assumed to be the same for all paths)

$$\mathbf{R}_\beta(n) = E[\boldsymbol{\beta}_k\boldsymbol{\beta}_{k-n}^H] = \varphi_n\mathbf{I}_d; \quad (10)$$

- Diversity model:

$$\mathbf{T} = [\Omega_1^{1/2}\mathbf{g}(\tau_1), \cdots, \Omega_d^{1/2}\mathbf{g}(\tau_d)] \otimes \mathbf{I}_{MN}, \quad (11)$$

with

$$\mathbf{R}_\beta(n) = \mathbf{R}_\beta\varphi_n \quad (12)$$

where $\mathbf{R}_\beta = \text{diag}\{\mathbf{R}_1, \mathbf{R}_2, ..., \mathbf{R}_d\}$ is block diagonal and the $MN \times MN$ matrix $\mathbf{R}_i$ accounts for the spatial fading correlation for the ith path at the transmitter and receiver side.

C. Sketch of the derivation of the lower bound

The signal model (7) can be restated in terms of the known matrix $\mathbf{F} = \bar{\mathbf{X}}\mathbf{U}$ as

$$\mathbf{y}_k = \mathbf{F}\mathbf{d}_k + \mathbf{n}_k. \quad (13)$$

The optimum (MMSE) channel estimator for $K \to \infty$ reduces to the MMSE estimation of the amplitudes $\mathbf{d}_k$. According to the (theoretically) infinite temporal horizon of our framework, we consider a Wiener filter that estimates the amplitudes in the frequency domain:

$$\mathcal{F}\{\hat{\mathbf{d}}_k\} = \mathbf{S}_{dy}(\omega)\mathbf{S}_{yy}(\omega)^{-1}\mathcal{F}\{\mathbf{y}_k\} \quad (14)$$

where $\mathbf{S}_{dy}(\omega) = \mathcal{F}\{E[\mathbf{d}_k\mathbf{y}_{k-n}^H]\}$ denotes the discrete-time Fourier transform of the crosscorrelation matrix between $\{\mathbf{d}_k\}$ and $\{\mathbf{z}_k\}$, $\mathbf{S}_{yy}(\omega)$ is similarly defined. Computing the performance of this estimator, we get the bound (4).

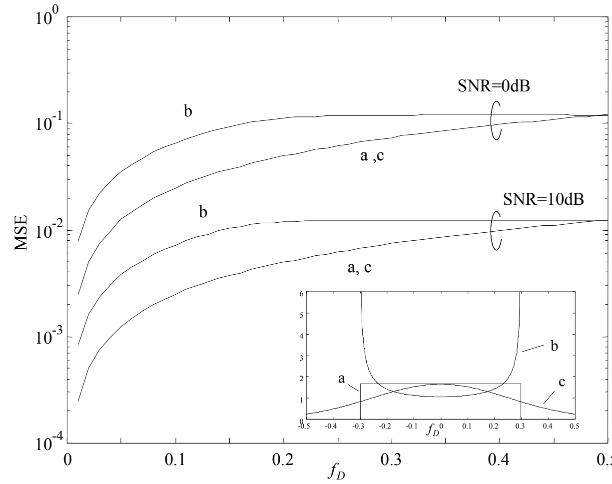

Fig. 1. MSE bound versus the Doppler spread f_D for the beamforming model ($M = N = 4, W = 8, d = 4, L = 32$).

III. Effect of System and Channel Parameters on the Lower Bound

In the following, we present numerical evaluations of the bound (4) in order to asses the impact of physical parameters on the channel estimation performance. Where not stated otherwise, the simulation setting is as follows: $M = N = 4$ transmitting and receiving antennas, $W = 8$, $d = 4$, the delays are integer multiples of the symbol rate, $\tau_i/T = i - 1$, and the angles are equally spaced in the angular support $[-60, 60]$ deg as $\alpha_i^{(T)} = \alpha_i^{(R)} = -60 + 120/d \cdot (i-1)$ deg. The noise is assumed to be spatially uncorrelated $\mathbf{Q} = \sigma_n^2\mathbf{I}_M$, the correlation matrix of training sequences is ideal, $\mathbf{R}_x = L\sigma_x^2\mathbf{I}_{NW}$, and the power-delay profile $\mathbf{\Omega}$ is scaled so that $E[||\mathbf{H}_k||^2] = M$. The length of the training sequences L is chosen as the minimum that allows a well conditioned LS channel estimation (from model (7)) ,i.e., $L = 32$. The SNR is defined as $SNR = \sigma_x^2/\sigma^2$. For the beamforming model, the arrays at both ends are assumed to be uniform linear with half-wavelength inter-element spacing.

A. Impact of Doppler spectrum on channel estimation

The lower bound (4) depends on the Doppler spectrum $S_\varphi(\omega)$. Here we evaluate the effect of different Doppler spectra on the channel estimation error. Fig. 1 shows the MSE bound as a function of the maximum Doppler spread f_D (normalized to the symbol rate $1/T$) for uniform (a), Clarke (b) and truncated Gaussian (for the latter case f_D represents the 3dB cut-off frequency) spectra. The beamforming model is considered but the results are qualitatively applicable to the diversity model as well. The spectra are scaled so that they correspond to equal transmitted power. Uniform and Clarke spectra lead to essentially the same performance in terms of MSE bound. On the other hand, in case the truncated Gaussian spectrum is more appropriate to model the fading correlation,

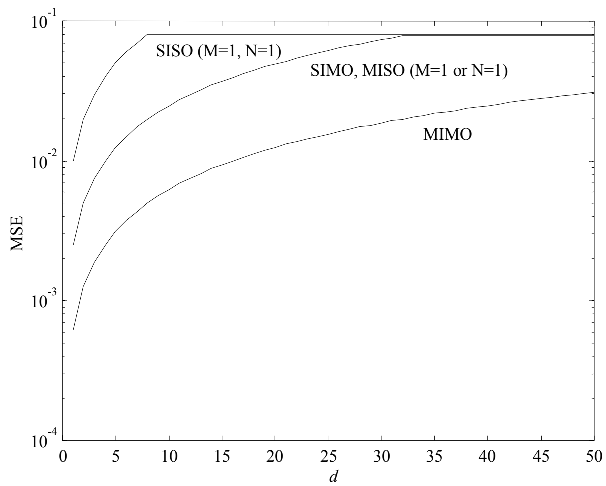

Fig. 2. MSE bound for different approaches to MIMO channel estimation vs. number of paths d for the beamforming model (uniform Doppler spectrum, $M = N = 4, W = 8, d = 4, L = 32, SNR = 0dB, f_D = 0.1$).

a degradation is expected. This can be explained by recalling that the estimator performs MMSE filtering of the fading amplitude which is more effective for fading variations with small bandwidth. In fact, for totally uncorrelated amplitudes within successive time-slots, MMSE filtering does not lead to any improvement in the channel estimation error. Accordingly, channel estimation performance for all spectra degrades for increasing channel variabilty, i.e., increasing Doppler shift.

B. MIMO vs. suboptimal approaches to channel estimation

Estimating all the frequency-selective channels from each transmit to each receive antenna simultaneously is theoretically the optimum way of performing MIMO channel estimation. On the other hand, for large M, N and W this task can become prohibitive. Therefore, instead of performing channel estimation by *jointly* considering all the (frequency-selective) SISO channels corresponding to each pair transmitting-receiving antenna, one could use sub-optimum approaches that estimates separately the SISO or the MISO/SIMO links. These approaches are detailed below:

- SISO approach: estimate separately the MN SISO channels corresponding to each pair transmitting-receiving antennas;
- MISO approach: estimate jointly all the N SISO channels relative to the links between all the transmitting antennas and one receiving antenna (M separate channel estimates)
- SIMO approach: estimate jointly all the M SISO channels relative to the links between one transmitting antennas and all the receiving antenna (N separate channel estimates).

Based on the discussion above, these approaches are just a special case of MIMO estimators. However, it is of interest here to evaluate the performance loss of these suboptimal approaches. From evaluation of the MSE bound for different values of M and N, it is possible to establish the expected degradation. In fact, if all the SISO channels share the same characteristics (as for transmitting and receiving antennas spaced not too far apart), the MSE bound for the SISO approach can be derived by specializing the MSE bound for $M = N = 1$ (with obvious notation):

$$MSE_{SISO} = MN \cdot MSE_{\hat{\mathbf{h}}}|_{M=1,N=1}. \tag{15}$$

On the other hand, the MSE bound on the performance of the SIMO (or dually MISO) approach to estimate the MIMO channel can be similarly obtained by evaluating the MSE bound for $N = 1$ for each of the M SIMO channels composing the MIMO link:

$$MSE_{SIMO} = N \cdot MSE_{\hat{\mathbf{h}}}|_{N=1}. \tag{16}$$

For high SNR, it can be shown using the results in [2] that for the beamforming model MSE_{SISO} is proportional to MNr_g and $MSE_{\hat{\mathbf{h}}}$ to r so that

$$\frac{MSE_{SISO}}{MSE_{\hat{\mathbf{h}}}} = \frac{MNr_g}{r} \tag{17}$$

where $r_g \le \min(d, W)$ is the number of resolvable delays, i.e., the number of linearly independent columns in matrix $[\mathbf{g}(\tau_1), \cdots, \mathbf{g}(\tau_d)]$, whereas $r \le \min(d, MNW)$ is the number of resolvable paths in the space-time domain, i.e., the number of linearly independent columns in matrix $\mathbf{T}$ (9). For well-resolved paths (in time), parameter r_g equals $\min(d, W)$ so that for $d \ge W$ r_g is constant and equal to W. Similiarly, for $d \ge MNW$ parameter $r = MNW$. It follows that for d large enough, there is no practical advantage in estimating jointly the MIMO channel with respect to the suboptimal SISO approach. Notice that for the diversity model it is easy to show that the SISO approach shows no perfomance degradation as compared to the MIMO approach.

The same reasoning can be applied to the SIMO approach (or dually to the MISO approach) noticing that for the beamforming model

$$\frac{MSE_{SIMO}}{MSE_{\hat{\mathbf{h}}}} = \frac{N \cdot r_{SIMO}}{r} \tag{18}$$

where $r_{SIMO} \le \min(d, MW)$ is the number of resolvable paths in the space (at the receiver side) - time domain, i.e., the number of independent columns in matrix $[\mathbf{g}(\tau_1) \otimes \mathbf{a}_R(\alpha_1^{(R)}), \cdots, \mathbf{g}(\tau_d) \otimes \mathbf{a}_R(\alpha_d^{(R)})]$.

Fig. 2 shows the MSE bound evaluated for the optimum (MIMO) approach and for the SISO, SIMO and MISO approaches versus the number of paths d for the beamforming model. As expected from the discussion above, the MSE of the SISO approach degrades until the number of paths d (and consequently r_g) equals $W = 8$ and similarly the MSE of the SIMO (or MISO) approach reaches a constant value for $d = MW = 32$ (or $d = NW = 32$). In a dense multipath environment (large number of paths d, in this case $d \ge MNW = 120$, not shown here), the degradation of these

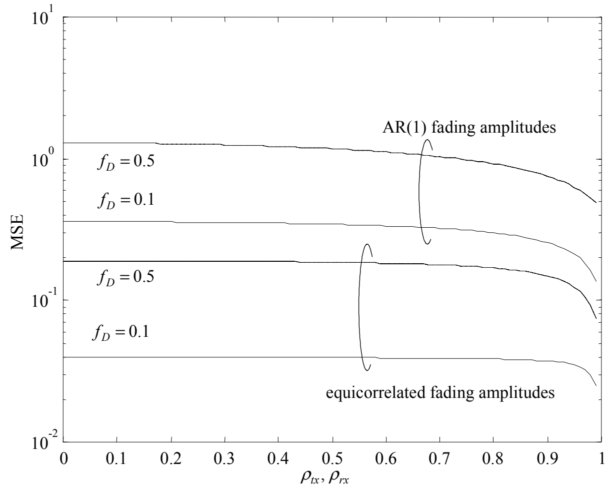

Fig. 3. MSE bound vs. the spatial correlation ρ_{tx},ρ_{rx} for the diversity model and two different spatial correlation models for the fading amplitudes (uniform Doppler spectrum, $M = N = 4, W = 8, d = 4, L = 32, SNR = 0dB, f_D = 0.1$)

suboptimum methods with respect to the MIMO approach decreases (to zero) showing that the advantage in performing full MIMO estimation has to be evaluated for the specific propagation channel.

C. Impact of spatial correlation of fading on channel estimation

Spatial correlation of fading amplitudes at the transmit and receive side of a MIMO link (diversity model) is known to have a remarkable impact on the system performance [3]. Here we study the effects of spatial correlation on channel estimation. For simplicity, the $MN \times MN$ spatial correlation matrices of all the paths $\mathbf{R}_i$ are assumed to be equal, $\mathbf{R}_i = \mathbf{R}$, and separable [3], $\mathbf{R} = \mathbf{R}_{tx} \otimes \mathbf{R}_{rx}$, where the $N \times N$ matrix $\mathbf{R}_{tx}$ accounts for spatial correlation at the transmitter side and $\mathbf{R}_{rx}$ at the receiver side. We consider two different spatial correlation models: *i*) AR(1) model: the spatial correlation matrix at the transmit (and receive) side is a symmetric Toepliz matrix characterized by the first column $[1\ \rho_{tx} \cdots \rho_{tx}^{N-1}]$ (and $[1\ \rho_{rx} \cdots \rho_{rx}^{N-1}]$), where $0 \leq \rho_{tx}, \rho_{rx} \leq 1$ are the spatial correlation coefficients; *ii*) equicorrelated fading amplitudes with correlation coefficient at the transmitter ρ_{tx} (and ρ_{rx} at the receiver).

Fig. 3 shows the MSE bound versus $\rho_{tx} = \rho_{rx}$. Channel estimation benefits from increasing fading correlation since MMSE filtering of the fading amplitudes becomes more effective. It should be noticed that while channel estimation can capitalize on increased spatial correlation, the capacity of the link (with perfect knowledge of the channel at the receiver) under the same assumption decreases [3]. By studying the capacity of the link with imperfect channel state information the impact of both effects can be assessed on the system performance [2].

IV. CONCLUSION

A lower bound on the correlation matrix of the channel estimation error for MIMO systems has been derived and investigated for practical systems through numerical results in order to show the impact of Doppler spectrum, spatial correlation, number of multipaths, number of transmit/receive antenna on channel estimation performance. The bound holds for both single-carrier and multi-carrier (MIMO-OFDM) systems. Among the presented results, the degradation of suboptimal, and computationally less intensive, (SISO/SIMO/MISO) approaches to channel estimation for MIMO link has been studied, showing that for dense multipath channel the performance loss can be negligible.

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