Combined Linear Pre-Equalization and BLAST Equalization With Channel Correlation Feedback

O. Simeone, *Student Member, IEEE*, and U. Spagnolini, *Senior Member, IEEE*

***Abstract*—In this letter, we study linear pre-equalization in conjunction with BLAST equalization. Linear pre-equalization is based on the assumption that the channel state information available at the transmitter is limited to the second order statistics of channel and noise (long term channel state information or LT-CSI). Through simulations, we compare different design criteria for the pre-equalizer and show the advantages of the considered structure with respect to BLAST.**

***Index Terms*—BLAST, channel state information (CSI), multiple input multiple output (MIMO), pre-equalization.**

I. Introduction

Space-time codes [1] and BLAST equalization [2] are well-known techniques designed with the aim of exploiting the capacity of multiple input multiple output (MIMO) links in absence of channel state information (CSI) at the transmitter. On the other hand, the linear pre-equalization/equalization transceivers proposed in [3] are based on the assumption that the transmitter is provided with the instantaneous realization of the fading channel $\mathbf{H}$ (istantaneous CSI or, in short, I-CSI). While the first approach may be too restrictive in channels with realistic fading rates, the latter may be too optimistic especially in FDD systems with low rate feedback links. Therefore, recently some authors considered MIMO links with CSI at the transmitter limited to the channel and noise correlation matrices (long-term CSI or, in short, LT-CSI). In particular, it has been shown that the optimum linear pre-equalizer in terms of capacity [4], probability of error for orthogonal space-time codes and MMSE for a linear equalizer [6] under the assumption of LT-CSI at the transmitter consists of an eigenbeamformer. In other words, the linear pre-equalizer is built from the eigenvectors of the spatial correlation matrix of the (whitened) channel at the transmitter side. In this letter, we combine the optimum pre-equalizer with an unordered BLAST receiver (Fig. 1). We investigate the beneficial effects of pre-equalization based on LT-CSI at the transmitter and compare different strategies of power allocation over the eigenvectors through simulation.

II. Signal and Channel Model

We focus on a MIMO wireless link with M transmit and N receive antennas $(N \geq M)$. According to Fig. 1, after linear pre-equalization by the $M \times M$ matrix $\mathbf{F}$ and propagation through the $N \times M$ frequency-flat radio channel $\mathbf{H}$, the received signal can be written as

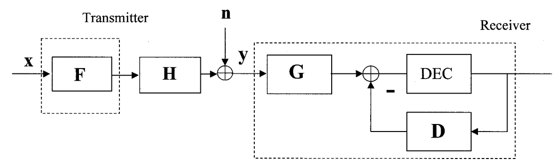

Fig. 1. Combined linear precoding and BLAST decoding.

$$\mathbf{y} = \mathbf{HFx} + \mathbf{n}, \tag{1}$$

where the circularly symmetric Gaussian noise has correlation $E[\mathbf{nn}^H] = \mathbf{R}_n$ and $\mathbf{x}$ is the $M \times 1$ transmitted vector, whose elements are taken from any constellation and $E[\mathbf{xx}^H] = \sigma_x^2\mathbf{I}$. Assuming that the noise spatial correlation is known (or can be reliably estimated from multiple observations), the receiver pre-processes the received vector $\mathbf{y}$ by whitening the additive noise $\mathbf{n}$ (the notation $(\cdot)^{1/2}$ defines the square root matrix $\mathbf{R} = \mathbf{R}^{H/2}\mathbf{R}^{1/2}$)

$$\tilde{\mathbf{y}} = \mathbf{R}_n^{-H/2} \quad \mathbf{y} = \tilde{\mathbf{H}}\mathbf{Fx} + \mathbf{w} \tag{2}$$

where $E[\mathbf{ww}^H] = \mathbf{I}$ and the equivalent (whitened) channel matrix reads $\tilde{\mathbf{H}} = \mathbf{R}_n^{-H/2}\mathbf{H}$.

The entries of the channel matrix $\mathbf{H}$ are assumed to be zero-mean circularly symmetric complex Gaussian (Rayleigh fading) with a separable spatial correlation function (see [7, Sect. II-B])

$$\mathbf{H} = \mathbf{R}_R^{H/2}\mathbf{H}_w\mathbf{R}_T^{1/2} \tag{3}$$

where $\mathbf{H}_w$ is a matrix of independent identically distributed Gaussian variables with unit power. The correlation matrices $\mathbf{R}_T$ and $\mathbf{R}_R$ account for the channel correlation at the transmit and receive side, respectively. The norm of the channel is $E[||\mathbf{H}||^2] = \mathrm{tr}\{\mathbf{R}_T\}\mathrm{tr}\{\mathbf{R}_R\} = MN$, where $E[\cdot]$ denotes the expectation over the fading amplitudes (for simplicity, $[\mathbf{R}_T]_{ii} = 1, i = 1, \ldots, M$ and $[\mathbf{R}_R]_{ii} = 1, i = 1, \ldots, N$).

III. Combined Linear Pre-Equalization and BLAST Equalization

Let the LT-CSI be available at the transmitter in the form of the matrix $\mathbf{R}_H = E[\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}]$. The LT-CSI matrix $\mathbf{R}_H$ measures the channel correlation at the transmitter side as it is easy to show the relationship

$$\mathbf{R}_H = \mathbf{R}_T \cdot \mathrm{tr}\left\{\mathbf{R}_n^{-1}\mathbf{R}_R\right\}. \tag{4}$$

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The authors are with Dipartimento di Elettronica e Informazione, Politecnico di Milano, 20133 Milano, Italy (e-mail: simeone@elet.polimi.it).

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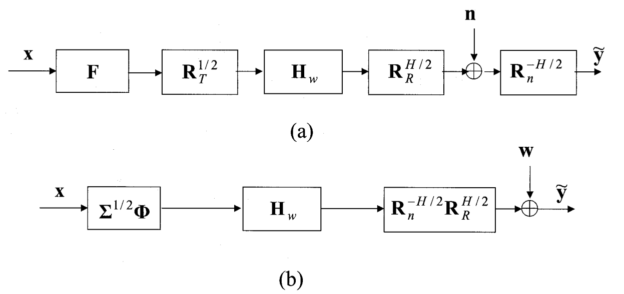

Fig. 2. (a) Block diagram of the signal model (transmitter, channel and noise whitener). (b) Equivalent signal model that shows the role of linear pre-equalization.

In particular, if we assume spatially white noise, i.e., $\mathbf{R}_n = \sigma_n^2\mathbf{I}$, it simplifies as $\mathbf{R}_H = N/\sigma_n^2\mathbf{R}_T$.

The linear pre-equalizer that maximizes the capacity [4], minimizes the probability of error for orthogonal space-time codes [5] and minimizes the MSE (between the transmitted vector $\mathbf{x}$ and the decision variables) for linear equalizers [6] under the assumption of LT-CSI at the transmitter has the following general expression:

$$\mathbf{F} = \mathbf{U\Phi} \tag{5}$$

where $\mathbf{U}$ contains the eigenvectors of the LT-CSI $\mathbf{R}_H = \mathbf{U\Lambda U}^H$ ($\mathbf{\Lambda} = \mathrm{diag}(\lambda_{11}, \ldots, \lambda_{MM})$ with λ_{ii} arranged in nonincreasing order) and $\mathbf{\Phi}$ =diag$(\phi_{11}, \ldots, \phi_{MM})$ is a diagonal matrix that defines the power allocation over the eigenvectors. The latter depends on the optimization criterion and the power constraint (see discussion below). According to (5), the linear pre-equalizer performs the beamforming of the transmitted vector $\mathbf{x}$ along the eigenvectors $\mathbf{U}$ of the LT-CSI matrix. Referring to the specific references for details, here we recall that the result (5) is obtained by bounding the average of the performance criterion (be it capacity, probability of error or MMSE) over the fading channel $\mathbf{H}$ and then optimizing the bound.

It is worth emphasizing that the correlation matrix $\mathbf{R}_H$, or its eigenvalue decomposition $\{\mathbf{U}, \mathbf{\Lambda}\}$, has to be updated only occasionally at the transmitter, e.g., by a low rate feedback channel in a FDD link, since it is assumed to be invariant over a large time scale. Notice that temporal variations of $\mathbf{R}_H$ (or equivalently $\mathbf{R}_T$) are likely to be caused by the movements of the transmitting array and these can be assumed to be small enough (compared to the geometry of environment and arrays) to guarantee the invariance of LT-CSI across multiple symbols.

According to (5), the role of the linear pre-equalizer (beamformer) $\mathbf{F}$ is that of cancelling the correlation at the transmitter side. Decorrelating the transfer matrix is known to guarantee enhanced link performance [7]. Fig. 2(a) shows the signal model (transmitter, channel and noise whitening) by explicitly including the channel correlation matrices. The cascade of $\mathbf{F}$ and $\mathbf{R}_T^{1/2}$ is a diagonal matrix since $\mathbf{R}_T = \mathbf{U\Sigma U}^H$ (with $\mathbf{\Sigma} = \mathbf{\Lambda}/\mathrm{tr}\{\mathbf{R}_n^{-1}\mathbf{R}_R\}$) and

$$\mathbf{R}_T^{1/2}\mathbf{F} = \mathbf{\Sigma}^{1/2}\mathbf{U}^H\mathbf{U\Phi} = \mathbf{\Sigma}^{1/2}\mathbf{\Phi}.$$

Therefore, the signal model can be simplified as in Fig. 2(b), proving by this simple reasoning that the correlation matrix at the transmitter side is diagonalized by the precoder $\mathbf{F}$. The equivalent channel correlation at the receiver side, $\mathbf{R}_n^{-H/2}\mathbf{R}_R^{H/2}$ has to be dealt with at the receiver.

We consider three different power allocation schemes over the eigenvectors that constraint the total radiated power as $E[||\mathbf{Fx}||^2] = \sigma_x^2\mathrm{tr}\{\mathbf{FF}^H\} = \sigma_x^2$.

A. Waterfilling

Maximizing the capacity [4] or minimizing the probability of error for orthogonal space-time codes [5] we get the classical waterfilling solution

$$|\phi_{ii}|^2 = \left(\frac{\sigma_x^2 + \sum_{n=1}^{\bar{N}}\lambda_{nn}^{-1}}{\sigma_x^2\bar{N}} - \frac{1}{\lambda_{ii}\sigma_x^2}\right)^+ \tag{6}$$

where $(x)^+ = \max(x, 0)$ and $\bar{N} \leq N$ is such that $|\phi_{nn}|^2 > 0$ for $n \in [1, \bar{N}]$ and $|\phi_{nn}|^2 = 0$ for all other n.

B. MMSE Waterfilling

Minimizing the MSE [6], [3] yields

$$|\phi_{ii}|^2 = \left(\frac{\sigma_x^2 + \sum_{n=1}^{\bar{N}}\lambda_{nn}^{-1}}{\sigma_x^2\sum_{n=1}^{\bar{N}}\lambda_{nn}^{-1/2}}\lambda_{ii}^{-1/2} - \frac{1}{\lambda_{ii}\sigma_x^2}\right)^+. \tag{7}$$

C. Uniform Power Allocation

Assuming that the same power is transmitted from each eigenmode

$$\phi_{ii} = \sigma_x/\sqrt{N}. \tag{8}$$

At the receiver side, the BLAST equalizer is designed here by minimizing the MSE between the data vector $\mathbf{x}$ and the input of the decision device (MMSE-BLAST [8]). Perfect error recovery at the output of the decision device is assumed as it is usually done in the literature on decision feedback equalization. The effect of error propagation will be investigated in Section IV through simulations. The MSE reads $\mathrm{MSE}(\mathbf{G}, \mathbf{E}) = E[||\mathbf{Gy} - \mathbf{Ex}||^2]$, where $\mathbf{E} = \mathbf{I} + \mathbf{D}$ is upper triangular. From the standard theory of Wiener linear filtering, we get $\mathbf{G} = \mathbf{E}E[\mathbf{xy}^H]E[\mathbf{yy}^H]^{-1}$ and after algebraic manipulations

$$\mathbf{G} = \mathbf{EQ}^{-1}\mathbf{F}^H\tilde{\mathbf{H}}^H\mathbf{R}_n^{-H/2}, \tag{9}$$

where $\mathbf{Q} = \mathbf{F}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F} + 1/\sigma_x^2\mathbf{I}$. Substituting (9) we obtain $\mathrm{MSE}(\mathbf{G}, \mathbf{E}) = \mathrm{tr}\{\mathbf{EQ}^{-1}\mathbf{E}^H\}$. Minimizing w.r.t. the feedback matrix $\mathbf{E}$ yields

$$\mathbf{E} = \mathbf{VQ}^{1/2} \tag{10}$$

where $\mathbf{V}$ is a diagonal matrix that scales to unity the elements on the main diagonal of $\mathbf{E}$.

IV. Simulation Results

The performance of combined linear pre-equalization and BLAST equalization is evaluated in terms of uncoded SER (averaged over the M transmitted streams) for a 16-QAM constellation and $N = M = 8$ antennas. Notice that the scheme could be modified in order to allow the transmission of

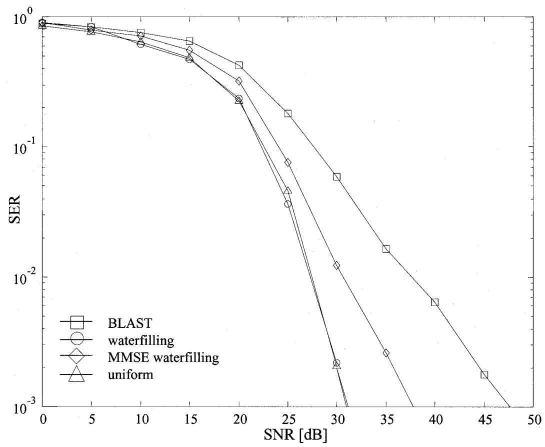

Fig. 3. SER versus SNR for LT-CSI ($\rho_T = 0.8, \rho_R = 0.4$ and $N = M = 8$).

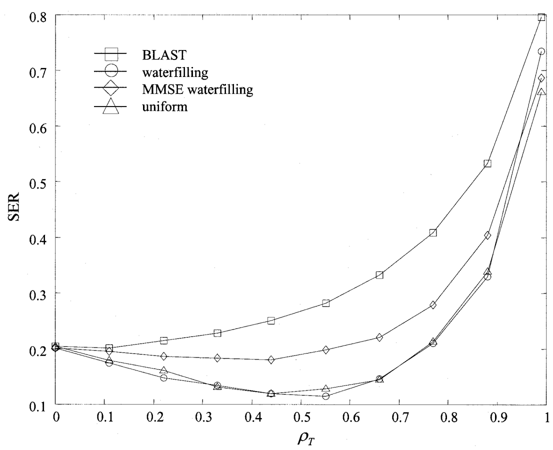

Fig. 4. SER versus ρ_T ($\mathrm{SNR} = 20$ dB, $M = N = 8, \rho_R = 0.4$).

$D < M$ streams (e.g., on the strongest eigenmodes). Different power allocation schemes under the assumption of LT-CSI at the transmitter are compared without optimal ordering of the data streams (unordered BLAST). We assume spatially white noise and the signal to noise ratio is defined as $\mathrm{SNR} = \sigma_x^2/\sigma_n^2$. The correlation matrix $\mathbf{R}_R$ (and $\mathbf{R}_T$) is for simplicity Toeplitz with first column $[1 \ \rho_R \ \cdots \ \rho_R^{N-1}]^T$ (and $[1 \ \rho_T \cdots \rho_T^{N-1}]^T$). ρ_R and $\rho_T (0 \leq \rho_R, \rho_T \leq 1)$ are the correlation coefficient relative to the receiver and transmitter side, respectively.

Fig. 3 shows the uncoded SER versus SNR for $\rho_T = 0.8$ and $\rho_R = 0.4$ and Fig. 4 plots the uncoded SER versus ρ_T for $\mathrm{SNR} = 20$ dB and $\rho_R = 0.4$ (analogous behavior as a function of ρ_T is obtained at lower SER as well). Linear pre-equalization with LT-CSI at the transmitter guarantees better performance with respect to BLAST over the entire range of correlation ρ_T and SNR considered. In particular, for $\mathrm{SER} = 10^{-3}$, pre-equalization with waterfilling provides around 15-dB gain in SNR. Moreover, waterfilling power allocation only slightly outperforms the uniform power allocation, that has the advantages of simplicity and reduced feedback (only the eigenvectors have to be transmitted by the receiver). Note that if the constellation size was allowed to vary with allocated power then waterfilling solution could potentially increase its SNR margin as compared to the uniform power scheme. The degradation of the MMSE waterfilling scheme (around 5 dB for $\mathrm{SER} = 10^{-3}$) can be explained by recalling that according to [6] the corresponding design assumes a linear equalizer (not a BLAST receiver).

For $\rho_T = 0$, the transmitter can not capitalize on the LT-CSI $\mathbf{R}_H = N/\sigma_n^2 \mathbf{I}$ to improve the performance of the link ($\mathbf{F} = \mathbf{I}$) so that BLAST and linear pre-equalization have the same SER. For large ρ_T the performance of all the methods degrades because of the decreased spatial diversity and the performance gap among the methods shrinks as the channel matrix $\mathbf{H}$ becomes rank-deficient.

V. Conclusion

Combined BLAST equalization and linear pre-equalization under the assumption of LT-CSI at the transmitter has been proposed. Different design criteria for the pre-equalizer have been investigated. Simulations showed the relevant advantages of pre-equalization with LT-CSI as compared to BLAST receiver.

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