Modal analysis/filtering to estimate time-varying MIMO-OFDM channels

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***Abstract*— In the multipath multiple-input multiple-output (MIMO) channel, long-term/large-scale properties (e.g., delays, angles, powers) remain approximately constant over many coherence times (i.e., over many Doppler periods) and define the structural terms of the propagation channel referred as delay (powers and delays) and angular (powers and angles of departure and arrival) modes of the channel. Effective improvement of the channel estimate is obtained by estimating delay and angular modal spaces via modal analysis and by filtering the (original) least squares channel estimates as projection onto the modal spaces. The paper also provides experimental validation of the overall method with realistic channel models.**

I. Introduction

MULTI-ANTENNA systems at both transmitter and receiver (MIMO) are currently being intensively investigated due to their ability to provide higher throughput and/or improved performance as compared to conventional single-input single-output (SISO) systems. If the multipath (SISO or MIMO) channel is frequency-selective (i.e., for large channel bandwidth), OFDM transmission can be used to mitigate its effects by counteracting intersymbol interference. With OFDM (applying K subcarriers) the frequency-selective channel is transformed into K parallel non-interfering channels. For these reasons MIMO-OFDM is an interesting candidate for 4G and next generation WLAN (e.g., IEEE 802.11n).

Accurate channel estimation at the receiver is mandatory to reliably detect the information data. Thus, the channel gains over the K subcarriers for $N_R N_T$ links have to be estimated (with N_T, N_R being the number of transmit and receive antennas, respectively). The problem is usually tackled with a pilot-based approach, transmitting on certain subcarriers known training symbols from appropriately designed training sequences across subcarriers and transmitting antennas [1] [2]. This paper presents a technique, which extends to MIMO-OFDM systems the subspace technique proposed in [3], to improve substantially the estimate of the channel. The technique, referred to as *modal analysis/filtering*, is based on the exploitation of the structural properties of time-varying MIMO channels. Delays, powers, and angles (departure/arrival) of the paths in the multipath MIMO channel are constant over a time scale that is much larger than the time scale for which the channel changes (i.e., the coherence time) and define the delay and angular modes of the channel. An improved estimate of the channel can then be obtained by projecting the least squares (LS) estimate onto the modal spaces, which are separately estimated. Validation of the technique through simulations with realistic channel model [4] is also provided.

Other techniques, as in [5] and [6], are based on the same idea of exploiting long-term invariance of the channel and reducing the number of parameters to be estimated in order to improve the accuracy in channel estimation. However, their parametric approach suffers from high computational complexity and thresholding effects due to non linear parameter estimation. The proposed unstructured approach, based on subspace filtering of the LS channel estimate, is also considered by [8] and [9], which capitalize respectively only on delay modes and angular modes.

The outline of the paper is as follows. The description of the system and of the space-time MIMO channel model are in Section II. Section III considers the channel estimation problem. First, LS channel estimation for MIMO-OFDM systems is reviewed in Section III-A, then the proposed modal analysis (MA) technique is introduced in Section III-B. In Section IV performance results in terms of Mean Square Error (MSE) are derived for the LS and MA estimation approaches to appreciate the gain provided by the latter. Experimental validation of the results is provided in Section V, first considering a simplified channel model (V-A) and then considering the 3GPP/3GPP2 SCM (spatial channel modeling) MIMO channel model [4] (V-B). Section VI concludes the paper.

Notation: in the paper bold denotes vectors or matrices (as it will be clear from the context); uppercase is used for frequency-domain quantities to distinguish them from the corresponding time domain quantities, which are lowercase; $(\cdot)^T$ denotes transposition; $(\cdot)^H$ denotes hermitian transposition; $(\cdot)^\dagger$ denotes matrix pseudoinverse; $tr[\cdot]$ represents trace; $E[\cdot]$ represents expectation; $vec\{\cdot\}$ is the stacking operator; if $\mathbf{X}$ is a given $M \times 1$ vector, $diag(\mathbf{X})$ is the $M \times M$ diagonal matrix with the entries of $\mathbf{X}$ on the main diagonal; the ith entry of a given vector $\mathbf{X}$ is denoted by $X[i]$; $[\mathbf{X}]_{i,j}$ denotes the (i,j)th entry of the matrix $\mathbf{X}$; $\mathbf{I}_M$ denotes the $M \times M$ unitary matrix.

II. Models

A. System model

An OFDM-MIMO system with N_T transmitting and N_R receiving antennas is considered in this paper. The $K \times 1$ vector $\mathbf{X}_\ell^{n_T}$ contains the complex symbols to be transmitted on the K subcarriers during symbol ℓ from antenna n_T. The MIMO scheme (e.g., space-time block coding, beamforming,

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space-time trellis coding, spatial multiplexing) determines how these vectors are generated from the input bit stream. OFDM modulation is carried out at each transmitting antenna, by using a cyclic prefix to cope with fourier transform periodicities. Let the maximum temporal support of the channel vector between the n_Tth transmitting antenna and the n_Rth receiving antenna $\mathbf{h}_\ell^{n_R,n_T}$ (which includes the impulse responses of the transmitting and receiving filters) be W, the received signal can be written in matrix notation as

$$\mathbf{Y}_\ell^{n_R} = \sum_{n_T=1}^{N_T} diag(\mathbf{X}_\ell^{n_T})\mathbf{H}_\ell^{n_R,n_T} + \mathbf{N}_\ell^{n_R}. \quad (1)$$

$\mathbf{N}_\ell^{n_R}$ is a $K \times 1$ vector of white noise at the n_Rth receiving antenna with $E[\mathbf{N}_\ell^{n_R}\mathbf{N}_\ell^{n_R H}] = \sigma_n^2\mathbf{I}_K$ and $\mathbf{H}_\ell^{n_R,n_T}$ is a $K\times 1$ vector containing the K-point DFT of $\mathbf{h}_\ell^{n_R,n_T}$:

$$\mathbf{H}_\ell^{n_R,n_T} = \mathbf{F}\mathbf{h}_\ell^{n_R,n_T}, \quad (2)$$

where $\mathbf{F}$ is the $K \times W$ DFT matrix with elements $[\mathbf{F}]_{k,w} = 1/\sqrt{K}\exp(-j\frac{2\pi}{K}kw)$, for $k = 1,..,K$, $w = 1,..,W$. The channel $\mathbf{H}_\ell^{n_R,n_T}$ and the noise $\mathbf{N}_\ell^{n_R}$ are statistically independent.

B. Channel model

The time-varying multipath fading channel at the ℓth OFDM symbol associated to the (n_R, n_T)th link has a channel impulse response given by the combination of D paths:

$$h_\ell^{n_R,n_T}(\tau) = \sum_{d=1}^{D} \sqrt{\Omega_{d,\ell}}[\mathbf{A}_{d,\ell}]_{n_R,n_T} g(\tau - \tau_{d,\ell}), \quad (3)$$

where $\Omega_{d,\ell}$ is the power of the dth path, $[\mathbf{A}_{d,\ell}]_{n_R,n_T}$ is the (n_R, n_T)th element of the $N_R \times N_T$ matrix $\mathbf{A}_{d,\ell}$ which collects the overall gains of the dth path on the N_RN_T links between transmitter and receiver, incorporating fading effects and array responses at both sides, $\tau_{d,\ell}$ is the delay of the dth path and $g(\tau)$ is the equivalent transmission filter $g(\tau) = g_T(\tau)*g_R(\tau)$, $g_T(\tau)$ and $g_R(\tau)$ being respectively the transmitter pulse-shaping function and the receiver matched filter impulse response.

Let us now sample at a rate $1/T$ ($T = 1/B$, where B is the transmission bandwidth) the channel impulse response on the N_RN_T links to obtain the $N_R \times N_TW$ MIMO-FIR channel matrix $\bar{\mathbf{h}}_\ell = [\bar{\mathbf{h}}_\ell[1]\cdots\bar{\mathbf{h}}_\ell[W]]$, where $\bar{\mathbf{h}}_\ell[w]$ is the $N_R \times N_T$ matrix collecting the wth sample $h_\ell^{n_R,n_T}((w-1)T)$ of the channel impulse responses over the N_RN_T links. Using the model (3) this reduces to

$$\bar{\mathbf{h}}_\ell = \sum_{d=1}^{D} \sqrt{\Omega_{d,\ell}}\mathbf{g}(\tau_{d,\ell})^T \otimes \mathbf{A}_{d,\ell}, \quad (4)$$

where $\mathbf{g}(\tau_{d,\ell})$ is the $W \times 1$ vector containing the T-spaced samples of the delayed waveform $g(\tau - \tau_{d,\ell})$. By stacking the entries corresponding to a given sample index w, we get the $N_RN_T \times W$ MIMO channel matrix $\mathbf{h}_\ell = [vec\{\bar{\mathbf{h}}_\ell[1]\}\cdots vec\{\bar{\mathbf{h}}_\ell[W]\}]$:

$$\mathbf{h}_\ell = \boldsymbol{\mathcal{A}}_\ell\boldsymbol{\Omega}_\ell^{1/2}\mathbf{G}(\boldsymbol{\tau}_\ell)^T \quad (5)$$

where $\boldsymbol{\mathcal{A}}_\ell = [vec\{\mathbf{A}_{1,\ell}\}\cdots vec\{\mathbf{A}_{D,\ell}\}]$ is $N_RN_T \times D$, $\boldsymbol{\Omega}_\ell^{1/2} = diag\{[\sqrt{\Omega_{1,\ell}}\cdots\sqrt{\Omega_{D,\ell}}]\}$ and $\mathbf{G}(\boldsymbol{\tau}_\ell) = [\mathbf{g}(\tau_{1,\ell})\cdots\mathbf{g}(\tau_{D,\ell})]$ is a $W\times D$ matrix collecting the sampled delayed waveforms.

In the case considered here, the antennas are closely spaced to each other, so that we can apply the far-field approximation (i.e., assuming locally plane wave) and the scattering environment is such that there are a few rays from the transmitter to the receiver, each characterized by a direction of departure (DOD $\alpha_d^{(T)}$) and a direction of arrival (DOA $\alpha_d^{(R)}$). This model is widely used for outdoor scenarios and has been adopted for MIMO systems in [10]. The array response vector $\mathbf{a}_{d,\ell}^{(R)}$ (and $\mathbf{a}_{d,\ell}^{(T)}$) is the manifold vector associated to the DOAs (and DODs). Thus, the set of paths for the ℓth symbol is identified by sets of DOAs ($\boldsymbol{\alpha}_\ell^{(R)} = [\alpha_{1,\ell}^{(R)}\cdots\alpha_{D,\ell}^{(R)}]$), DODs ($\boldsymbol{\alpha}_\ell^{(T)} = [\alpha_{1,\ell}^{(T)}\cdots\alpha_{D,\ell}^{(T)}]$) and fading complex amplitudes ($\boldsymbol{\beta}_\ell = [\beta_{1,\ell}\cdots\beta_{D,\ell}]^T$), to give the $N_R \times N_T$ matrices $\mathbf{A}_{d,\ell} = \beta_{d,\ell}\mathbf{a}^{(R)}(\alpha_{d,\ell}^{(R)})\mathbf{a}^{(T)}(\alpha_{d,\ell}^{(T)})^T$, defined for each of the D paths.

So far we have considered the time-variability of the channel by using the subscript ℓ. However, the quantities in (5) are known to have a different degree of time-variability. In fact, the geometry and the characteristics of the scatterers are supposed to vary slowly compared to the coherence time of the channel. As a consequence, delays $\{\tau_{d,\ell}\}$, DOAs $\{\alpha_{d,\ell}^{(R)}\}$ and DODs $\{\alpha_{d,\ell}^{(T)}\}$ and powers in $\boldsymbol{\Omega}_\ell^{1/2}$can be considered as constant over multiple - say L - training OFDM symbols, thus losing their dependence on the subscript ℓ. On the other hand, the vector $\boldsymbol{\beta}_\ell \sim \mathcal{CN}(0, \mathbf{I}_D)$ (the fading on different paths is uncorrelated) accounts for the fast-fading variations of the path amplitudes and, as such, is variable on a much smaller time scale, typically on the order of the duration a mobile needs to travel a few to a fraction of the wavelength.

According to this assumption, time-varying model in (5) becomes

$$\mathbf{h}_\ell = \boldsymbol{\mathcal{A}}'(\boldsymbol{\alpha}^{(T)}, \boldsymbol{\alpha}^{(R)}) \cdot diag(\boldsymbol{\beta}_\ell) \cdot \boldsymbol{\Omega}^{1/2}\mathbf{G}(\boldsymbol{\tau})^T, \quad (6)$$

where $\boldsymbol{\mathcal{A}}_\ell$ has now been factorized into two terms, $\boldsymbol{\mathcal{A}}'(\boldsymbol{\alpha}^{(T)}, \boldsymbol{\alpha}^{(R)}) = [\mathbf{a}^{(T)}(\alpha_1^{(T)})\otimes\mathbf{a}^{(R)}(\alpha_1^{(R)}), \cdots, \mathbf{a}^{(T)}(\alpha_D^{(T)})\otimes \mathbf{a}^{(R)}(\alpha_D^{(R)})]$ and $diag(\boldsymbol{\beta}_\ell)$, that sepatate the terms that depend on the (slowly-varying) angles $\boldsymbol{\alpha}^T, \boldsymbol{\alpha}^R$ and on the (fast-varying) fading coefficients $\boldsymbol{\beta}_\ell$

Considering the powers Ω_d normalized to have $\sum_{d=1}^{D}\Omega_d = 1$, it can be shown that

$$E\left[|H_\ell^{n_R,n_T}[k]|^2\right] = \frac{1}{K} \quad ,\forall k, \forall \ell, \quad (7)$$

where $H_\ell^{n_R,n_T}[k]$ is the kth entry of the channel vector $\mathbf{H}_\ell^{n_R,n_T}$ defined in (2). As a consequence, it is convenient to define the signal to noise ratio (SNR) per frequency bin as

$$SNR = \frac{E\left[|Y_\ell^{n_R}[k]|^2\right]}{E\left[|N_\ell^{n_R}[k]|^2\right]} = \frac{E\left[|H_\ell^{n_R,n_T}[k]|^2\right] \cdot E\left[\|\mathbf{X}_\ell[k]\|^2\right]}{\sigma_n^2} = \frac{1}{K}\frac{P}{\sigma_n^2}, \quad (8)$$

where $P = E\left[\|\mathbf{X}_\ell[k]\|^2\right]$ is the total power transmitted on a given subcarrier by all the N_T transmitting antenna, independent on k (for non-adaptive power allocation).

III. Channel estimation by modal analysis

A. Least Squares channel estimation

An effective way to estimate the channel for SISO/SIMO OFDM systems is to allocate K_p known pilot subcarriers according to some periodic pattern in the time-frequency grid. For a W-taps channel, $K_p \geqslant W$ properly selected pilot subcarriers are enough to get an estimate immune to aliasing [11]. For a MIMO system, the scenario becomes rather complicated due to the interference of the N_T simultaneous sources, which make the estimation of the channel from the received data more difficult. In order to estimate the channel over the $N_R N_T$ links during an OFDM symbol, at least WN_T pilot tones have to be used for training, whereas the remaining tones are used to transmit information data. Thus, $WN_T \leqslant K_p N_T \leqslant K$ is necessary to get an estimate considering only one OFDM training symbol (if the channel can be considered costant over several OFDM symbols, the $K_p N_T$ can be distributed in different OFDM symbols [1]). In the following we analyse the case when $K_p N_T$ pilots are used for each training OFDM symbol and only the subcarrier indices where training symbols are inserted are considered.

LS channel estimation is carried out independently for each element (n_R) of the array at receiver side. The MISO model (1) can be written, without the subscript ℓ for clarity of notation, with respect to the channel impulse responses $\mathbf{h}^{n_R,n_T}$:

$$\begin{aligned}\bar{\mathbf{Y}}^{n_R} &= \sum_{n_T=1}^{N_T} diag(\bar{\mathbf{X}}^{n_T})\bar{\mathbf{H}}^{n_R,n_T} + \bar{\mathbf{N}}^{n_R} = \\ &= \left[diag(\bar{\mathbf{X}}^1)\bar{\mathbf{F}}...diag(\bar{\mathbf{X}}^{N_T})\bar{\mathbf{F}}\right]\left[\mathbf{h}^{n_R,1T}..\mathbf{h}^{n_R,N_T T}\right]^T + \bar{\mathbf{N}}^{n_R} = \\ &= \mathbf{B}\mathbf{h}^{n_R} + \bar{\mathbf{N}}^{n_R}\end{aligned} \quad (9)$$

where $\bar{\mathbf{Y}}^{n_R}$, $\bar{\mathbf{X}}^{n_T}$, $\bar{\mathbf{N}}^{n_R}$, $\bar{\mathbf{F}}$ are the matrices composed by the $K_p N_T$ rows of $\mathbf{Y}^{n_R}$, $\mathbf{X}^{n_T}$, $\mathbf{N}^{n_R}$, $\mathbf{F}$ corresponding to the pilot tones, $\mathbf{B} = \left[diag(\bar{\mathbf{X}}^1)\bar{\mathbf{F}} \cdots diag(\bar{\mathbf{X}}^{N_T})\bar{\mathbf{F}}\right]$ is the $K_p N_T \times WN_T$ matrix composed of the $K_p N_T \times W$ training matrices $diag(\bar{\mathbf{X}}^{n_T})\bar{\mathbf{F}}$ and $\mathbf{h}^{n_R} = \left[\mathbf{h}^{n_R,1T} \cdots \mathbf{h}^{n_R,N_T T}\right]^T$. Thus the LS estimate of $\mathbf{h}^{n_R}$ is

$$\mathbf{h}_{LS}^{n_R} = \mathbf{B}^\dagger \bar{\mathbf{Y}}^{n_R} \quad (10)$$

and it is unbiased as $\mathbf{h}_{LS}^{n_R} = \mathbf{h}^{n_R} + \mathbf{B}^\dagger \bar{\mathbf{N}}^{n_R}$.

Minimum MSE of the LS channel estimate can be obtained by using optimal training sequences $\bar{\mathbf{X}}^{n_T}$ across the pilot tones and the transmitting antennas [1] [2], which are are equipowered, equispaced and phase shift orthogonal. For each OFDM training symbol, $K_p N_T$ pilots are used simultaneously by all the N_T transmitting antennas:

$$\begin{aligned}\bar{X}^{n_T}[i] &= \left[diag(\bar{\mathbf{X}}^{n_T})\right]_{i,i} = \sqrt{\frac{P}{N_T}} \exp\left(-j\pi v_{n_T} i/K_p\right), \\ \forall i &= 1,..,K_p N_T,\ \forall n_T = 1,..,N_T,\end{aligned} \quad (11)$$

where P is the total power transmitted on a given subcarrier by the N_T transmitting antennas and v_{n_T} is chosen for any n_T to keep phase shift orthogonality between sequences associated to different transmitting antennas.

B. Modal Analysis/Filtering

The channel estimate can be improved exploiting the characteristics of the MIMO wireless channel. As observed in Sec. II-B, the multipath channel is given by the superposition of several delay-resolvable paths (e.g., denoted macropaths in [4]), each characterized by a given power, delay and DOA/DOD. Possibility for improvement of the channel estimate is at hand if we exploit the correlation of the channel coefficients over space (i.e., over the total of $N_T N_R$ links) and the reduced number of relevant taps (as compared with the maximum length W) of the channel impulse response by identifying the angular and delay modes of the channels. Reduction of the error in the estimate can be obtained when the number of angular channel modes (i.e. the resolvable paths in the angular domain, determined by the DOAs/DODs of the different paths and by the aperture of the antenna arrays) is smaller than $N_R N_T$ or if the number of delay channel modes (i.e., the resolvable paths in the delay domain, determined by the delays of the paths and by the bandwidth of the system) is smaller than W.

Modal filtering of the LS estimate exploits stationarity of the large-scale properties far beyond the time scale for which the channel impulse response (or channel transfer function) can be assumed to be constant (i.e., coherence time). In particular, the geometry and the characteristics of the scattering environment are supposed to vary slowly compared to the fast (small-scale) fading, which varies on a much smaller time scale (i.e., coherence time). The stationary delays, DOAs and DODs of the paths that compose the channel impulse response define the angular modes (in $\boldsymbol{\mathcal{A}}'(\boldsymbol{\alpha}^{(T)}, \boldsymbol{\alpha}^{(R)})$) and delay modes (in $\boldsymbol{\Omega}^{1/2}\mathbf{G}(\boldsymbol{\tau})^T$) of the channel. Effective improvement of the channel estimates is obtained by estimating angular and delay modal spaces via modal analysis and filtering of the (original) LS channel estimates. This is obtained by projecting the LS estimate onto the modal spaces, reducing in this way the error in the estimates. The approach is convenient for its unstructured nature and does not suffer problems due to non-convex objective function in parameter estimation and high computational complexity.

The channel matrix $\mathbf{h}_\ell$ in (6) can be written with the alternative parametrization [12]

$$\mathbf{h}_\ell = \mathbf{U}_S \boldsymbol{\Gamma}_\ell \mathbf{U}_T^H, \quad (12)$$

that can eventually be derived from the singular value decomposition of $\boldsymbol{\mathcal{A}}'(\boldsymbol{\alpha}^{(T)}, \boldsymbol{\alpha}^{(R)}) = \mathbf{U}_S \boldsymbol{\Sigma}_S \mathbf{V}_S^H$, $\mathbf{G}(\boldsymbol{\tau})\boldsymbol{\Omega}^{1/2} = \mathbf{U}_T \boldsymbol{\Sigma}_T \mathbf{V}_T^H$ and by definition of $\boldsymbol{\Gamma}_\ell = \boldsymbol{\Sigma}_S \mathbf{V}_S^H diag(\boldsymbol{\beta}_\ell)\mathbf{V}_T \boldsymbol{\Sigma}_T^H$. The estimates of the modes $\hat{\mathbf{U}}_S$ (and $\hat{\mathbf{U}}_T$) are obtained taking the principal r_S (and r_T) eigenvectors of the spatial and temporal covariance

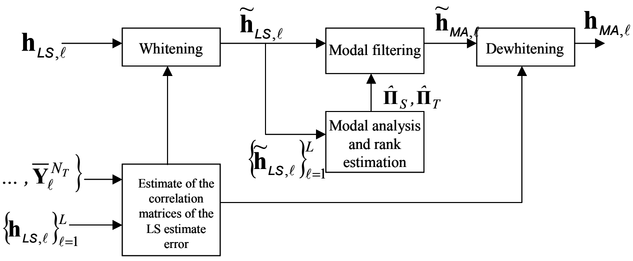

Fig. 1. Block diagram of the modal analysis/filtering technique. Whitening and dewhitening need to be performed if the error over the initial LS estimate is not spatially or temporally uncorrelated.

matrices:

$$\hat{\mathbf{U}}_S = eig_{r_S}\left\{\frac{1}{L}\sum_{\ell=1}^{L}\mathbf{h}_{LS,\ell}\mathbf{h}_{LS,\ell}^H\right\} \quad (13a)$$

$$\hat{\mathbf{U}}_T = eig_{r_T}\left\{\frac{1}{L}\sum_{\ell=1}^{L}\mathbf{h}_{LS,\ell}^H\mathbf{h}_{LS,\ell}\right\}. \quad (13b)$$

An improved channel estimate, referred to as MA estimate $\mathbf{h}_{MA,\ell}$, is then derived by filtering the LS channel estimate $\mathbf{h}_{LS,\ell}$ based on the knowledge of the spatial and temporal modes:

$$\mathbf{h}_{MA,\ell} = \hat{\mathbf{\Pi}}_S\mathbf{h}_{LS,\ell}\hat{\mathbf{\Pi}}_T, \quad (14)$$

where $\hat{\mathbf{\Pi}}_S = \hat{\mathbf{U}}_S\hat{\mathbf{U}}_S^H$ and $\hat{\mathbf{\Pi}}_T = \hat{\mathbf{U}}_T\hat{\mathbf{U}}_T^H$ are the projectors onto the space and time modal spaces.

The dimensions of the angular and delay modal spaces are $r_S = rank\left[\boldsymbol{\mathcal{A}}'(\boldsymbol{\alpha}^{(T)}, \boldsymbol{\alpha}^{(R)})\right]$ and $r_T = rank\left[\mathbf{G}(\boldsymbol{\tau})\mathbf{\Omega}^{1/2}\right] = rank\left[\mathbf{G}(\boldsymbol{\tau})\right]$. These determined by the number of resolvable paths in the angular and temporal domains. The model orders r_S and r_T are upper bounded by the number of antenna pairs $N_R N_T$ and by the temporal support of the channel W: $r_S < \min(D, N_R N_T)$ and $r_T < \min(D, W)$.

Channel estimation could be more accurate by taking into account jointly delays and angles, as in [13], but here we limit to the disjoint analysis because of its reduced computational cost and its faster convergence in estimation of the modes.

The proposed modal filtering technique is applicable for LS estimates affected by uncorrelated estimation error. In this case the contribution of the noise is equally spread over the dimensions of the modal spaces and then effective improvement in estimation can be achieved by selecting only the principal (signal) components of the corresponding spaces. Thus, some modifications are to be intoduced to the MA channel estimation in order to cope with correlated estimation error. Error over the initial LS channel estimate can be correlated due to spatially colored noise at the receiver or to correlation among training sequences over time and transmitting antennas (e.g., when Decision Directed estimation takes place, the symbols used to estimate the channel are not orthogonal over transmitting antennas and subcarriers indices). In this case whitening of the LS estimate has to be performed before modal analysis and filtering take place. A detailed discussion on the topic is given in [14]. Fig. 1 illustrates the overall scheme of the modal analysis/filtering technique.

Tracking mode implementation [14] performs tracking of the modal subspaces and adaptive channel rank estimation on a symbol basis. This approach, that can be easily adapted from [3], counteracts the flaws inherent in the batch implementation presented here, i.e., the latency of L training symbols in providing the MA channel estimate and the inability to track slow variations of the channel modes, which are characteristics of the mobile radio channel.

IV. MSE PERFORMANCE ANALYSIS

Given the LS estimate of the channel impulse responses for all the (n_R, n_T) links $\mathbf{h}_{LS,\ell}$ stacked into the $N_R N_T W \times 1$ vector $\boldsymbol{h}_{LS,\ell} = vec\{\mathbf{h}_{LS,\ell}\}$, the $N_R N_T K \times 1$ vector $\boldsymbol{\mathcal{H}}_{LS,\ell}$, containing the frequency-domain channel estimate on the K subcarriers, is obtained by DFT transform of $\boldsymbol{h}_{LS,\ell}$

$$\boldsymbol{\mathcal{H}}_{LS,\ell} = (\mathbf{F}\otimes\mathbf{I}_{N_R N_T})\,\boldsymbol{h}_{LS,\ell} = \boldsymbol{\mathcal{H}}_\ell + (\mathbf{F}\otimes\mathbf{I}_{N_R N_T})\,\boldsymbol{\varepsilon}_{LS,\ell}, \quad (15)$$

where $\boldsymbol{\varepsilon}_{LS,\ell}$ is the $N_R N_T W \times 1$ vector containing the error on the LS estimate of $\boldsymbol{h}_\ell$ ($\boldsymbol{h}_{LS,\ell} = \boldsymbol{h}_\ell + \boldsymbol{\varepsilon}_{LS,\ell}$). If optimum training sequences (11) are used, then $\boldsymbol{\varepsilon}_{LS,\ell} \sim \mathcal{CN}(0, \frac{1}{SNR\cdot K_p}\mathbf{I}_{N_R N_T W})$, being $SNR = \frac{1}{K}\frac{P}{\sigma_n^2}$ (8).

Considering the MA approach, we can write the channel estimate as a projection of the LS estimate onto the angular and delay channel modes

$$\boldsymbol{\mathcal{H}}_{MA,\ell} = (\mathbf{F}\otimes\mathbf{I}_{N_R N_T})\,\boldsymbol{h}_{MA,\ell} \xrightarrow{L\to\infty} \rightarrow \boldsymbol{\mathcal{H}}_\ell + (\mathbf{F}\otimes\mathbf{I}_{N_R N_T})\,(\mathbf{\Pi}_T\otimes\mathbf{\Pi}_S)\,\boldsymbol{\varepsilon}_{LS,\ell}, \quad (16)$$

where the result relies on the assumption of consistency: for $L \to \infty$ $\hat{\mathbf{\Pi}}_S \to \mathbf{\Pi}_S$ and $\hat{\mathbf{\Pi}}_T \to \mathbf{\Pi}_T$ [12]. The following property of the kronecker product was used: $vec\{\mathbf{ABC}\} = \left(\mathbf{C}^T\otimes\mathbf{A}\right)vec\{\mathbf{B}\}$. Thus, $vec\left\{\hat{\mathbf{\Pi}}_S\mathbf{h}_{LS,\ell}\hat{\mathbf{\Pi}}_T\right\} = \left(\hat{\mathbf{\Pi}}_T\otimes\hat{\mathbf{\Pi}}_S\right)\boldsymbol{h}_{LS,\ell}$.

From (15) the MSEs of the LS estimate $\boldsymbol{\mathcal{H}}_{LS,\ell}$ is

$$MSE_{\boldsymbol{\mathcal{H}},LS} = \frac{1}{SNR\cdot K_p}\left\|\mathbf{F}\otimes\mathbf{I}_{N_R N_T}\right\|^2 = \frac{N_R N_T W}{SNR\cdot K_p}, \quad (17)$$

Using equation (16) we get the asymptotic ($L \to \infty$) MSE expression for the Modal Analysis approach:

$$MSE_{\boldsymbol{\mathcal{H}},MA} = \frac{1}{SNR\cdot K_p}E\left[tr\Big[\hat{\mathbf{\Pi}}_S\hat{\mathbf{\Pi}}_S^H\Big]tr\Big[\mathbf{F}\hat{\mathbf{\Pi}}_T\hat{\mathbf{\Pi}}_T^H\mathbf{F}^H\Big]\right]\xrightarrow{L\to\infty} \rightarrow \frac{r_S r_T}{SNR\cdot K_p}. \quad (18)$$

We can appreciate the improved performances of the method based on modal analysis over the standard LS estimation, with a gain of $\frac{N_R N_T W}{r_S r_T}$ due to the exploitation of the characteristics of the channel model in (6), with a spatial gain of $\frac{N_R N_T}{r_S}$ and a temporal gain of $\frac{W}{r_T}$. These results are justified by the reduction in the numbers of parameters to estimate in the factorized channel model (6): from $N_R N_T$ to r_S in the spatial/angular domain and from W to r_T in the time/delay domain, because of the limited number of resolvable paths in the respective domains.

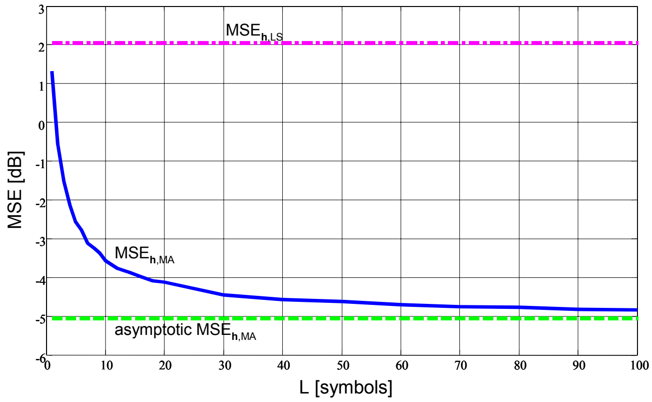

Fig. 2. $MSE_{\mathcal{H},MA}(L)$ with $SNR = 10dB$

We can also write the correlation matrix of the error of the MA channel estimate as

$$\mathbf{R}_{\varepsilon\ MA} = \frac{1}{SNR \cdot K_p} E\left[\left(\mathbf{F}\hat{\mathbf{\Pi}}_T \otimes \hat{\mathbf{\Pi}}_S\right)\left(\hat{\mathbf{\Pi}}_T \mathbf{F}^H \otimes \hat{\mathbf{\Pi}}_S\right)\right] \overset{L\to\infty}{\longrightarrow} \to \frac{1}{SNR \cdot K_p}\left(\mathbf{F}\mathbf{\Pi}_T\mathbf{F}^H \otimes \mathbf{\Pi}_S\right). \quad (19)$$

V. Simulation results

A. Simplified MIMO channel

The performance of the proposed channel estimation method is evaluated through simulations for an OFDM system with $K = 32$ subcarriers, $N_T = 4$ transmitting antennas, $N_R = 4$ receiving antennas. The time-varying channel is generated accordingly to the model presented in Section II-B. It is given by the superposition of $D = 5$ temporally resolved paths and has a temporal support $W = 8$. The number of pilot tones for each transmitting antenna is $K_p = W = 8$ and, as a consequence, a total of $N_T K_p = 32$ subcarriers are used for training (then in such a system there would be no information data transmitted within a training symbol). The paths are characterized by DODs equally-spaced in $(20°, 80°)$ and DOAs equally-spaced in $(110°, 170°)$ (hence, the dimension of the angular modal space is $r_S = \min(D, N_R N_T) = 5$) and have different sample-spaced delays (hence, the dimension of the delay modal space is $r_T = \min(D, W) = D = 5$).

The channel is considered to be invariant over L OFDM training symbols w.r.t. angles, delays and powers of the paths. L OFDM training symbols with uncorrelated fading amplitudes ($\mathbf{R}_\beta(n) = E\left[\boldsymbol{\beta}_\ell \boldsymbol{\beta}_{\ell-n}^H\right] = \mathbf{I}_D \cdot \delta(n)$) are then used to estimate the angular and delay modes. The result in (18) is valid asymptotically (for $L \to \infty$) and hence the greater L the closer the actual result will be to the analytic expressions. The perfomance as a function of L is evaluated in fig. 2 for a fixed SNR=10dB. The MSE of the LS estimate and the asymptotic MSE with the MA approach are shown for reference. We notice that the performance converges quite rapidly with L. Then, $L = 20$ is chosen for following simulations as a good trade-off between complexity and accuracy.

The gain of the Modal Analysis approach over the standard LS estimation when the dimensions of the spatial and temporal modal spaces have been accurately defined is in fig. 3, where there is the dependence of the MSE vs. the selected spatial filtering dimension $\widehat{r_S}$. For large SNRs, underestimating the dimension of the modal spaces (i.e., obtaining a biased estimate) means discarding part of the signal energy and it leads to a performance loss. On the contrary, for small SNRs a choice of $\widehat{r_S} < r_S$ is effective in filtering out a larger fraction of the noise, which is equally spread across the dimensions of the modal space. The minimum MSE for high SNR (SNR=20dB) is obtained for $\widehat{r_S} = r_S = 5$ (unbiased estimate). For low SNR, it is beneficial to accept a biased estimate, the minimum MSE being obtained for $\widehat{r_S} < r_S$ ($\widehat{r_S} = 1$ for SNR=-10dB). Instead, for intermediate SNRs, the MSE curve w.r.t. the chosen subspace dimension $\widehat{r_S}$ is rather flat around $\widehat{r_S} = r_S$ (see curve for SNR=5dB).

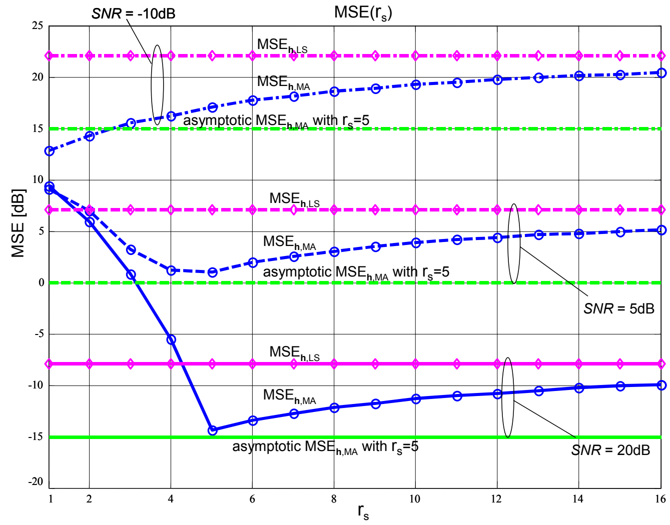

Fig. 3. $MSE_{\mathcal{H},MA}(\widehat{r_S})$ with different values of SNR

B. 3GPP/3GPP2 SCM

The modal filtering technique is tested with a MIMO channel model based on the work [4] in the 3GPP/3GPP2 SCM (spatial channel modeling) adhoc group, defined to be used as a common reference for evaluating different MIMO concepts. It is a channel model where the propagation between transmitter (BS) and mobile receiver (MS) is modelled by several macropaths, characterized by delays, powers and mean angles of departure/arrival and each composed by 20 subpaths. Power-delay profile and angles of departure/arrival are stocastically generated accordingly to the specifications in [4] and related to the chosen environment scenario. The results were obtained considering the *suburban macrocell* scenario, which is the one which provides smallest angular spread at the BS. The MIMO-OFDM system considered is equipped at both sides with 4-elements ($N_R = 4$, $N_T = 4$) half-wavelength spaced linear antenna arrays. The bandwidth of 5MHz (hence, the bit period with BPSK modulation is $T = 200nsec$), centered in 1.9 GHz, is divided into $K = 128$ subcarriers. A maximum delay of $6, 4\mu sec$ is assumed so that $K_p = 32$ pilot subcarriers can be reserved (if some delay of the multipath

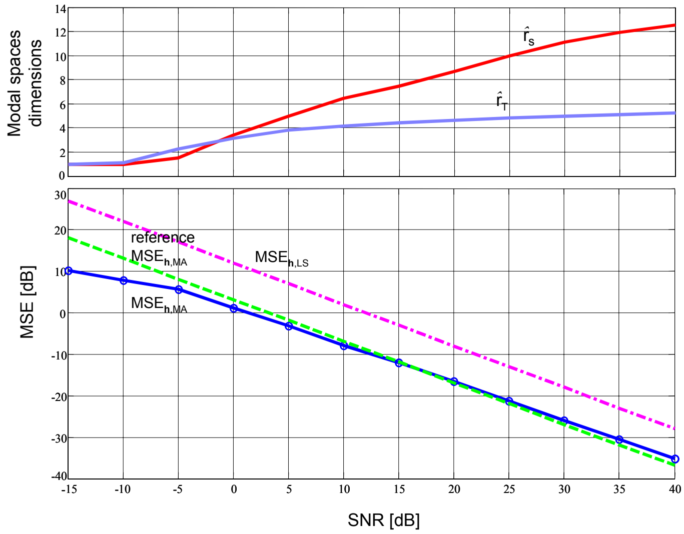

Fig. 4. MSE_{MA}, MSE_{LS} and estimated dimensions r_S, r_T of the modal spaces as a function of the SNR

channel exceeds $6,4\mu sec$, then we incur estimation error due to aliasing in the frequency domain). A speed of $54\ km/h$ has been considered for the MS. Training symbols have been inserted every $N = 10$ OFDM symbols. $L = 200$ training OFDM symbols are considered for estimating the channel modal spaces.

As the number of resolvable paths is not pre-defined in the SCM channel model, rank estimation has to select the number of significant modes of the channel. We observed in Section V-A the trade-off between bias (due to the discard of some signal dimensions) and variance (due to the noise present in the selected dimensions) of the estimation error. The selection of the filtering dimensions for the MA technique is performed, as in [9], to minimize an objective function given by the sum of bias and variance errors. The optimal filtering spatial (temporal) dimensions are estimated by comparing the eigenvalues λ_i of the spatial (temporal) covariance matrices in (13) with the noise contribute to these eigenvalues, related to the power of the noise σ_n^2 (assumed to be known). Thus, we minimize the overall error due to bias and noise influence. Fig. 4 shows the MSE of the MA and LS channel estimates and the selected filtering dimensions r_S, r_T of the modal spaces for different SNRs. Besides, a reference MSE curve is shown, derived applying (18) to the modal dimensions estimated for high SNR (here $40dB$). The actual MSE of the MA channel estimate outperforms this result as it selects for every SNR the filtering dimensions which minimize the overall estimation error.

VI. Conclusion

A channel estimation technique for MIMO-OFDM systems which profits of the different degree of time variability between slowly-varying channel modes (identified by DODs/DOAs, powers and delays of the paths) and fast-varying fading amplitudes has been presented. We showed that significant improvement in the estimate of the channel can be obtained when the number of relevant angular and delay modes is smaller than the maximum number of parameters to be estimated for a MIMO channel in the spatial and temporal domain. Performance results have been presented, which showed the dependence of the MSE on the number of angular and delay modes of the channel. Moreover, simulation with channel models proved the benefits provided also with realistic MIMO channels. Tracking mode implementation of the MA allows effective employment of the technique for channel environments with slow variations of the modes [14].

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