Adaptive pilot pattern for OFDM systems

O. Simeone and U. Spagnolini
Dip. di Elettronica e Informazione, Politecnico di Milano
P.zza L. da Vinci, 32 I-20133 Milano (Italy), e-mail: {simeone,spagnoli}@elet.polimi.it

***Abstract*— In OFDM communication systems over fading channels, link adaptation based on the available channel state information at the transmitter (i.e., power allocation, adaptive modulation and coding rate) increases the spectral efficiency and reliability of the link.**

In this work, we explore the possibility to extend the set of transmission parameters to be adaptively selected to the pilot arrangement (i.e., to the collocation of pilot subcarriers on the time-frequency grid). Based on the prediction of the channel estimation error at the receiver, the transmitter can minimize the number of pilot subcarriers that guarantees a sufficiently reliable channel estimate according to Quality of Service requirements. Prediction of the channel estimation error is performed by computing the error covariance matrix of Kalman filters.

The adaptive pilot arrangement problem is formulated and a "greedy" solution is proposed. Simulations show the effectiveness of the algorithm with respect to periodic re-training.

I. Introduction

In order to achieve high spectral efficiency and reliability, wireless data transmission over fading channels requires techniques that are able to adaptively adjust to the channel state. Based on the channel state information (CSI) available at the transmitter, transmission parameters such as transmit power, constellation size and coding scheme can be adaptively chosen at the physical layer in order to satisfy some quality of service (QoS) criterion [1]. In an OFDM system, transmission parameters can be selected independently on a subcarrier or, more practically, group of subcarriers basis [2].

Most of the work on link adaptation has assumed perfect CSI (see, e.g., [1]). Channel estimation errors have been taken into account by assessing how reliable the estimate has to be in order to assure "negligible" performance degradation [3] [4]. In an OFDM system, channel estimation is usually performed by sending training (or pilot) symbols on subcarriers (pilot subcarriers) known at the receiver [5]. The quality of the channel estimate depends on the pilot arrangement, i.e., on the number of pilot subcarriers and on their collocation in the time-frequency grid.

In this work we propose to extend the set of parameters to be adapted based on CSI to the pilot arrangement. As with the other adaptive transmission techniques, adaptive pilot placement can greatly increase the spectral efficiency of the system. QoS requirements from higher layers determine a maximum channel estimation error that the system can tolerate. Based on the prediction of channel estimation error at the receiver, the transmitter can allocate the pilot pattern over a given number of OFDM symbols with the goal of guaranteeing the reliability of the channel estimate while minimizing the number of pilot subcarriers.

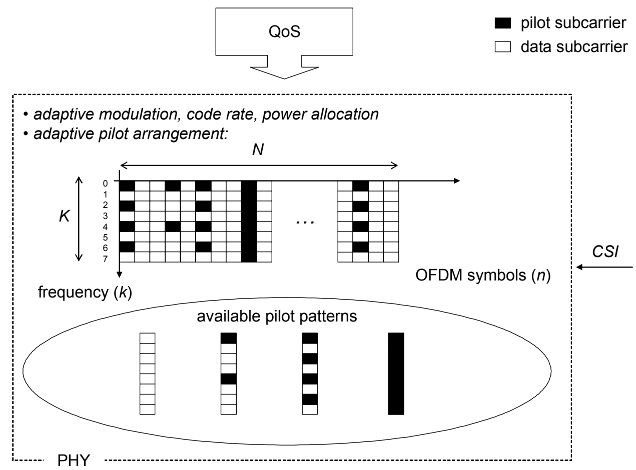

Fig. 1. System description.

Prediction of the channel estimation error at the transmitter is performed by considering the performance of a Kalman channel estimator at the receiver. The transmitter is assumed to known the second order statistics of the channel and the average signal to noise ratio. This CSI can be either acquired directly by the transmitter in a TDD link or fedback by the receiver in a FDD link. Notice that in the latter case, the feedback link is only required to support a low rate since the considered CSI can be assumed to be slowly varying (long term CSI).

II. System description

A. Adaptive pilot pattern: motivation and fundamentals

The system is illustated in fig. 1. The physical layer of the link employs OFDM modulation with training-based channel estimation. Higher layers set some constraint on QoS, such as frame/symbol error probability. The physical layer then adapts transmission parameters (modulation, code rate and power) in order to satisfy the QoS requirements. Furthermore, it computes a maximum signal to noise (SNR) degradation η due to channel estimation (or equivalently a maximum channel estimation error) that leads to negligible system performance loss. A certain SNR degradation can be guaranteed by appropriately placing pilot subcarriers in each OFDM symbol as it will be explained in the following. The adaptive choice of the pilot pattern is made every N OFDM symbols, where N can be for instance the duration of the time-slot assigned to a given user in a TDMA system [5]. The total number of subcarriers

 0-7803-8533-0/04/$20.00 (c) 2004 IEEE

is K. Notice that K could be a fraction of the subcarriers available in the bandwidth of an OFDMA system [6].

To ease the implementation of the OFDM modulator, the number of available subcarriers K is set to be a power of 2, $K = 2^l$. The subcarriers are indexed as $k = 0, 1, ..., K-1$ while the OFDM symbols are denoted as $n = 0, 1, ..., N-1$ (see fig. 1). Equi-spaced pilot subcarriers in the frequency domain are known to minimize the channel estimation error [7]; accordingly, the number of pilot arrangements that constitute possible choices for the transmitter in each OFDM symbol are $l+1$ and precisely (fig. 1):

1. no pilot subcarriers ($K_p = 0$): the OFDM symbol contains only data subcarriers;

2. $K_p = 2^m$ equispaced pilot subcarriers (with $m = 1, ..., l$): the pilot subcarriers occupy the frequency bins $i \cdot 2^{l-m}$ with $i = 0, 1, ..., K_p - 1$.

B. OFDM modulation and channel model

A detailed description of an OFDM link can be found in, e.g., [8]. Here we recall that the signal received on the kth sucarrier in the nth OFDM symbol can be written as

$$y_k(n) = H_k(n)x_k(n) + w_k(n), \quad (1)$$

where $H_k(n)$ is the channel gain and $w_k(n)$ the additive Gaussian noise. By stacking the signal received over the used bandwidth ($\mathbf{y}(n) = [y_0(n)\ y_1(n) \cdots y_{K-1}(n)]^T$), the measurement within the nth OFDM symbol can be written as

$$\mathbf{y}(n) = \mathbf{X}(n)\mathbf{H}(n) + \mathbf{w}(n), \quad (2)$$

with $\mathbf{X}(n)$ denoting a diagonal matrix defined by the vector of transmitted symbol $\mathbf{x}(n) = [x_0(n)\ x_1(n) \cdots x_{K-1}(n)]^T$ and $\mathbf{H}(n) = [H_0(n)\ H_1(n) \cdots H_{K-1}(n)]^T$ representing the channel vector in the frequency domain. The additive noise is uncorrelated in time and frequency, $E[\mathbf{w}(n)\mathbf{w}(n-t)^H] = \sigma^2\mathbf{I}\boldsymbol{\delta}(t)$ ($\boldsymbol{\delta}(t) = 0$ for $t \neq 0$ and $\boldsymbol{\delta}(t) = 1$ for $t = 0$). As it is generally assumed (see, e.g., [9]), the channel $\mathbf{H}(n)$ is taken to be the Fourier transform of the $L \times 1$ channel vector in the time domain $\mathbf{h}(n)$ (L is the temporal support of the channel)

$$\mathbf{H}(n) = \mathbf{F}\mathbf{h}(n), \quad (3)$$

where $\mathbf{F}$ is the $K \times L$ FFT matrix ($F_{k\ell} = 1/\sqrt{K}\exp(-i2\pi/K \cdot k\ell)$ with $k = 0, 1, ..., K-1$ and $\ell = 0, 1, ..., L-1$). The L entries of the channel vector $\mathbf{h}(n)$ are assumed to be independent circularly symmetric complex Gaussian variables with zero mean and power delay profile defined by a diagonal matrix $\mathbf{D}$, that is

$$\mathbf{h}(n) = \mathbf{D}^{1/2}\mathbf{g}(n), \quad (4)$$

where $\mathbf{g}(n)$ is a $L \times 1$ vector of independent circularly symmetric Gaussian variables with zero mean and $E[\mathbf{g}(n)\mathbf{g}(n)^H] = \mathbf{I}$. The temporal correlation of the channel is assumed to be well approximated over the N OFDM symbols by an AR(1) model, i.e., $E[\mathbf{g}(n)\mathbf{g}(n-t)^H] = \rho^t\mathbf{I}$, where ρ can be written as a function of the Doppler shift f_D as $\rho = J_o(2\pi f_D T)$ (T is the duration of an OFDM symbol) [12]. Notice that the second order statistics of the channel

$$E[\mathbf{h}(n)\mathbf{h}(n-t)^H] = \mathbf{D}\boldsymbol{\rho}^t \quad (5)$$

are assumed to be constant over the temporal horizon of interest (N OFDM symbols). Moreover, the channel is scaled so that $E[||\mathbf{h}(n)||^2] = \sum_{\ell=0}^{L-1} D_\ell = 1$.

From (3) and (4), the channel correlation in the frequency domain is $\mathbf{R}_H = E[\mathbf{H}(n)\mathbf{H}(n)^H] = \mathbf{F}\mathbf{D}\mathbf{F}^H$ or equivalently for the entries $R_{H,ij} = \sum_{\ell=0}^{L-1} F_{i\ell}F_{j\ell}^* D_\ell$, which implies that: a) unless $L = K$ and the power delay profile is uniform (e.g., $D_\ell = D$ for $\ell = 0, 1, ..., L-1$), the channel is correlated over frequency; b) the channel power in each frequency bin is the same: $R_{H,kk} = E[|H_k(n)|^2] = 1/K \cdot \sum_{\ell=0}^{L-1} D_\ell = 1/K$.

C. Effective (average) signal to noise ratio SNR_{eff}

The average SNR over the kth subcarrier and the nth OFDM symbol in absence of channel estimation error can be written as

$$\begin{aligned} SNR_k(n) &= (E[|H_k(n)|^2]E[|x_k(n)|^2])/\sigma^2 = \\ &= E[|x_k(n)|^2]/(K\sigma^2). \end{aligned} \quad (6)$$

The average SNR (6) depends on the frequency bin k and on the OFDM symbol n if adaptive power allocation is employed on a subcarrier (or group of subcarrier) basis and/or on a symbol by symbol basis (i.e., if $E[|x_k(n)|^2]$ is a function of n and/or k). Here, we simplify the presentation by setting $E[|x_k(n)|^2] = \sigma_x^2$. The extension to the most general case of adaptive power allocation is not covered here as it is straightforward and requires minor modifications. In conclusion, the average SNR (6) in absence of channel estimation errors within the time-frequency bins allocated to the link under consideration is

$$SNR = \frac{\sigma_x^2}{K\sigma^2}. \quad (7)$$

The effect of channel estimation errors can be taken into account by defining an "effective" average SNR as follows. By introducing the channel estimate $\hat{H}_k(n)$, the received signal (1) can be restated as

$$y_k(n) = \hat{H}_k(n)x_k(n) + (H_k(n) - \hat{H}_k(n))x_k(n) + w_k(n). \quad (8)$$

The term related to the channel estimation error, $(H_k(n) - \hat{H}_k(n))x_k(n)$, can be then modelled as additive Gaussian noise with zero mean and power $E[|H_k(n) - \hat{H}_k(n)|^2]\sigma_x^2$, assuming that the error and the transmitted signal are independent [10]. For equi-spaced equi-powered pilot subcarriers, the channel estimate variance $E[|H_k(n) - \hat{H}_k(n)|^2] = \sigma_e^2(n)$ is independent on k so that the effective SNR can be defined as

$$SNR_{eff}(n) = \frac{\sigma_x^2}{K(\sigma^2 + \sigma_e^2(n)\sigma_x^2)} = \frac{SNR}{1 + \bar{\sigma}_e^2(n)SNR}, \quad (9)$$

where $\bar{\sigma}_e^2(n)$ is the normalized (with respect to $E[|H_k(n)|^2]$) channel estimation error: $\bar{\sigma}_e^2(n) = \sigma_e^2(n)/(1/K)$.

D. Adaptive pilot pattern: problem formulation

As previously stated, the physical layer at the transmitter computes a maximum SNR loss η due to channel estimation. Equivalently, the physical layer sets a minimum effective SNR, denoted as $\overline{SNR}_{eff}$, that it is required to guarantee over the N OFDM symbols. Feasibility calls for $\overline{SNR}_{eff} < SNR$, or in other words $\eta = SNR/\overline{SNR}_{eff} > 1$.

The goal of the physical layer is to place pilot subcarriers over the N OFDM symbols in order to guarantee that

$$SNR_{eff}(n) \geq \overline{SNR}_{eff}, \tag{10}$$

or in terms of channel estimation error

$$\bar{\sigma}_e^2(n) \leq \frac{1}{\overline{SNR}_{eff}} - \frac{1}{SNR}. \tag{11}$$

In the next Section, we will be able to write $\bar{\sigma}_e^2(n)$ as a function of the pilot pattern for a Kalman channel estimator. The physical layer at the transmitter side tries to *guarantee the minimum effective SNR ($\overline{SNR}_{eff}$, see (10) or (11)) for $n = 0, 1, ..., N-1$ by using the minimum number of pilot subcarriers over the N OFDM symbols.*

Remarks: 1) Decision directed or iterative techniques can improve the channel estimation accuracy at the cost of increased complexity. When these techniques are implemented, the requirement on $\bar{\sigma}_e^2(n)$ can be alleviated accordingly.

2) Similarly to all the adaptive transmission techniques, adaptive pilot pattern requires the transmitter to inform the receiver about the selected pilot arrangement (over the $l+1$ available) for each n.

3) If the adaptive modulation and code rate algorithm has selected different modes for different subcarriers or OFDM symbols, the maximum allowable SNR loss could be a function of k and n itself. In this case, a possible choice in our framework is to define η with respect to the most demanding requirement, that is $\eta = \min_{n,k} \eta(n,k)$.

III. Kalman channel estimation

From (4), (5) and (2), the channel model and the received signal can be written in a state form equation

$$\mathbf{g}(n) = \rho\mathbf{g}(n-1) + \mathbf{v}(n) \tag{12a}$$

$$\mathbf{y}(n) = \mathbf{C}(n)\mathbf{g}(n) + \mathbf{w}(n), \tag{12b}$$

where $\mathbf{C}(n) = \mathbf{X}(n)\mathbf{F}\mathbf{D}^{1/2}$ and $\mathbf{v}(n)$ is zero mean circularly symmetric Gaussian with $E[\mathbf{v}(n)\mathbf{v}(n-t)^H] = \sqrt{(1-\rho^2)}\mathbf{I}\boldsymbol{\delta}_t$. Assuming that the second order statistics of the channel (long term CSI) ρ (or equivalently the Doppler shift f_D) and $\mathbf{D}$ are known at the receiver (e.g., from long term measurements of the channel), the $L \times 1$ vector $\mathbf{g}(n)$ can be tracked by a Kalman filter [11]. Since we focus on training-based channel estimation, that is we do not consider the data symbols as useful information for channel estimation, the vector $\mathbf{x}(n)$ will be thereafter redefined as $\tilde{\mathbf{x}}(n)$:

$$\tilde{x}_k(n) = 0 \quad \text{if the } k\text{th subcarrier contains a data symbol}$$

$$\tilde{x}_k(n) \neq 0 \quad \text{if the } k\text{th subcarrier contains a pilot symbol.}$$

Matrices $\tilde{\mathbf{X}}(n)$ and $\tilde{\mathbf{C}}(n) = \tilde{\mathbf{X}}(n)\mathbf{F}\mathbf{D}^{1/2}$ are defined accordingly.

The error correlation matrix $\mathbf{R}_g(n) = E[(\mathbf{g}(n) - \hat{\mathbf{g}}(n))(\mathbf{g}(n)-\hat{\mathbf{g}}(n))^H]$ of the Kalman filter can be written as a function of the pilot pattern $\tilde{\mathbf{x}}(n)$ following the standard theory [11]. Since the estimate of the frequency domain channel vector is then obtained as

$$\hat{\mathbf{H}}(n) = \mathbf{F}\mathbf{D}^{1/2}\hat{\mathbf{g}}(n), \tag{14}$$

the error correlation matrix in the frequency domain follows as

$$\begin{aligned}\mathbf{R}_e(n) &= E[(\mathbf{H}(n) - \hat{\mathbf{H}}(n))(\mathbf{H}(n) - \hat{\mathbf{H}}(n))^H] = \\ &= \mathbf{F}\mathbf{D}^{1/2}\mathbf{R}_g(n)\mathbf{D}^{1/2}\mathbf{F}^H. \end{aligned} \tag{15}$$

Notice that $\sigma_e^2(n) = R_e(n)_{ii}$. It is easy to show that $\mathbf{R}_e(n)$ can be computed recursively as a function of the pilot pattern $\tilde{\mathbf{x}}(n)$ as (see Appendix-A)

$$\begin{aligned}\mathbf{R}_e(n) &= \mathbf{A}(n) - \rho^2\mathbf{A}(n)\tilde{\mathbf{X}}(n)^H \cdot \\ &\cdot (\tilde{\mathbf{X}}(n)\mathbf{A}(n)\tilde{\mathbf{X}}(n)^H + \sigma^2\mathbf{I})^{-1}\tilde{\mathbf{X}}(n)\mathbf{A}(n) \end{aligned} \tag{16a}$$

$$\mathbf{A}(n) = \rho^2\mathbf{R}_e(n-1) + (1-\rho^2)\mathbf{R}_H. \tag{16b}$$

Equations (16a) and (16b) can be implemented at the transmitter as long as the long term CSI (i.e., ρ and $\mathbf{D}$) is known at the transmitter either by direct measurement (TDD link) or by (low rate) feedback by the receiver (FDD link). The initial estimation ($n = 0$) of the channel is assumed to be obtained by a traditional least squares estimate

$$\hat{\mathbf{g}}(0) = (\tilde{\mathbf{C}}(0)^H\tilde{\mathbf{C}}(0))^{-1}\tilde{\mathbf{C}}(0)^H\mathbf{y}(0), \tag{17}$$

so that $\mathbf{R}_g(0) = \sigma^2(\tilde{\mathbf{C}}(0)^H\tilde{\mathbf{C}}(0))^{-1}$ (see, e.g., [7]) and

$$\begin{aligned}\mathbf{R}_e(0) &= \sigma^2\mathbf{F}\mathbf{D}^{1/2}(\tilde{\mathbf{C}}(0)^H\tilde{\mathbf{C}}(0))^{-1}\mathbf{D}^{1/2}\mathbf{F}^H = \\ &= \sigma^2\mathbf{F}(\mathbf{F}^H\mathbf{D}_x(0)\mathbf{F})^{-1}\mathbf{F}^H, \end{aligned} \tag{18}$$

where the pilot pattern power profile $\mathbf{D}_x(n) = \tilde{\mathbf{X}}(n)^H\tilde{\mathbf{X}}(n)$ has been defined. Notice that in order to make the estimate (17) feasible, the initial training pattern $\tilde{\mathbf{x}}(0)$ has to contain at least $K_p \geq L$ pilot subcarriers.

Restating the problem illustrated in Sec. II-D, the transmitter has to guarantee that $\sigma_e^2(n) = R_e(n)_{ii}$ satisfies (11) for $n = 0, 1, ..., N-1$ while minimizing the number of pilot subcarriers.

A. Training sequence design

From (18) it is clear that the initial channel estimation error only depends on the pilot pattern power profile $\mathbf{D}_x(n)$: the training sequence $\tilde{\mathbf{x}}(n)$ can be drawn from any constellation as long as the power profile is the same. The same applies to the channel estimation error for any n as it is shown in Appendix-B. Moreover, here we are interested as for the derivation of (9) in a uniform channel estimation error over the subcarriers so that we opt for a uniform power allocation over the pilot subcarriers.

IV. "GREEDY" PILOT PATTERN

The choice of the optimum pilot pattern $\tilde{\mathbf{x}}(n)$ $n = 0, 1, ..., N-1$ would require to explore a large number of possible solutions, on the order of $(l+1)^N$. A suboptimum solution that proved to perform satisfactorily is the "greedy" algorithm described below:

for $n = 0$: select the minumum $m = 1, ..., l$ such that: a) $K_p = 2^m \geq L$ and b) the constraint (11) is satisfied;

for $n > 0$: select the minimum $K_p = 0$ or $K_p = 2^m$ ($m = 1, ..., l$) such that (11) is satisfied.

This algorithm is referred to as greedy since it selects for each OFDM symbol the best immediate solution, i.e., the minimum number of pilot subcarriers that guarantees (11).

V. SIMULATION RESULTS

Let us consider an OFDM link with $K = 64$ subcarriers and a block of $N = 10$ OFDM symbols. Moreover, $f_D T = 0.1$ ($\rho = 0.9975$), $L = 16$ and the power delay profile is uniform, $\mathbf{D} = 1/L\mathbf{I}$. We start with a simple example in order to clarify the system under study and to show the effectiveness of the greedy solution. The SNR is known to be $SNR = 20dB$ and the physical layer sets as a requirement a SNR loss $\eta = 5dB$ or equivalently $\overline{SNR}_{eff} = 15dB$. It follows from (11) that $\bar{\sigma}_e^2(n) \leq 0.0216$. The dashed line in fig. 3 represents this upper bound on the normalized error $\bar{\sigma}_e^2(n)$. A first choice for the pilot pattern could be to place in each OFDM symbol the same number of pilot K_p that allows the initial least squares estimate to guarantee $\bar{\sigma}_e^2(0) \leq 0.0216$. Fig. 2 shows this pattern in the first row ("fixed pattern") with $K_p = L = 16$. In this case, the total number of pilot subcarriers employed over $N = 10$ OFDM symbols turns out to be $16 \times 10 = 160$. If we used the initial least squares estimate for the entire block of N symbols ("only initial training"), we would end up with only 16 pilot subcarriers employed but we would not be able to satisfy (11) for $n > 4$. A practical and widely used choice is to re-train the system whenever we need it ("periodic re-training"). In this case, the use of $16 \times 4 = 64$ pilot subcarriers is enough to satisfy (11) for every n. As shown in fig. 2-3, the greedy pilot pattern not only satisfies the SNR requirement but also allows to reduce the number of pilot subcarriers to 52, thus improving the spectral efficiency of the system. Notice that letting N grow, we could notice that the greedy pilot pattern reaches a steady state periodic pilot arrangement with period of 6 OFDM symbols such that K_p over each period varies as $\{0, 0, 2, 8, 2, 16\}$. By constrast, the "periodic re-training" algorithm (optimized to guarantee the same SNR loss $\eta = 5dB$) has a period of 3 OFDM symbols with K_p varying as $\{0, 0, 16\}$.

We now consider the performance of the greedy adaptive pilot pattern compared to the traditional periodic re-training approach for varying channel correlation (fig. 4) and SNR (fig. 5) for two different SNR loss $\eta = 3dB$ (dashed lines) and $\eta = 5dB$ (solid lines). As in the previous example, both schemes are implemented so as to satisfy (10) for $n = 0, 1, ..., 9$. The performance is shown in terms of the fraction of total subcarriers ($N \times K = 10 \times 64$) used as pilot subcarriers. As expected, increasing the channel variability (i.e., $f_D T$) decreasing the SNR loss η or increasing SNR cause the number of needed pilot subcarriers to increase. Moreover, the greedy algorithm (circles) uniformly outperforms the periodic training scheme (triangles) leading to a more efficient use of the available bandwidth.

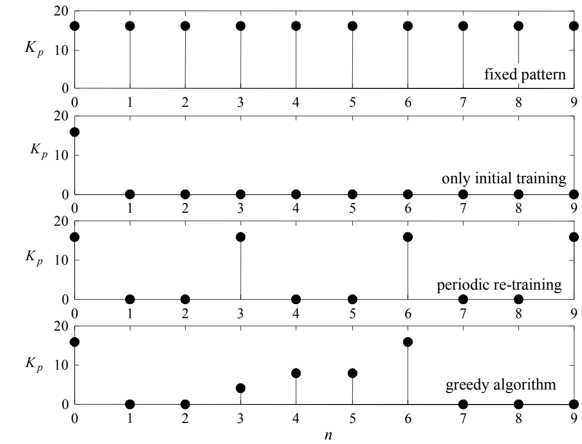

Fig. 2. Number of pilot subcarriers K_p vs. n for different pilot arrangements.

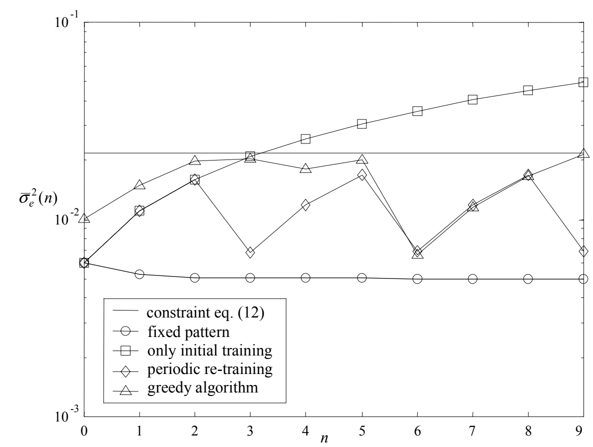

Fig. 3. Normalized channel estimation error $\bar{\sigma}_e^2(n)$ as a function of the OFDM symbol n for different pilot arrangements. Dashed line represents the constraint (11).

VI. CONCLUSION

Extension of link adaptability to the pilot pattern has been proposed. A suboptimum solution based on the greedy principle has been considered and its effectiveness evaluated through

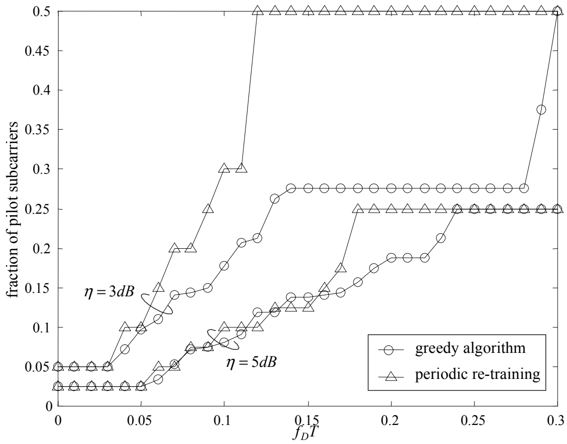

Fig. 4. Fraction of total subcarriers ($N \times K = 10 \times 64$) used as pilot subcarriers for the periodic re-training and greedy algorithm vs. the channel variability (Doppler) f_DT ($SNR = 20dB$).

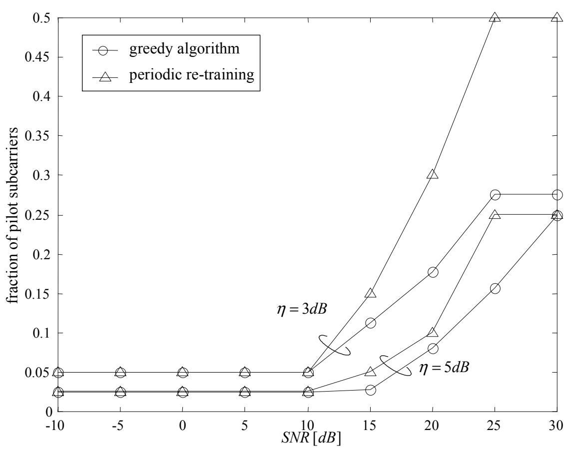

Fig. 5. Fraction of total subcarriers ($N \times K = 10 \times 64$) used as pilot subcarriers for the period re-training and greedy algorithm vs. SNR ($f_DT = 0.1$).

simulation with respect to known (non-adaptive) pilot arrangements. Our findings show that adaptive pilot arrangement can greatly improve the spectral efficiency of the system when compared to conventional strategies.

VII. Appendix-A

For Kalman filtering [11], the correlation matrix $\mathbf{R}_g(n)$ can be computed recursively as

$$\mathbf{R}_g(n) = \breve{\mathbf{R}}_g(n) - \rho \mathbf{G}(n)\tilde{\mathbf{C}}(n)\breve{\mathbf{R}}_g(n), \tag{19}$$

where

$$\breve{\mathbf{R}}_g(n) = \rho^2 \mathbf{R}_g(n-1) + (1-\rho^2)\mathbf{I} \tag{20}$$

is the correlation matrix of the one-step prediction error and

$$\mathbf{G}(n) = \boldsymbol{\rho}\breve{\mathbf{R}}_g(n)\tilde{\mathbf{C}}(n)^H(\tilde{\mathbf{C}}(n)\breve{\mathbf{R}}_g(n)\tilde{\mathbf{C}}(n)^H + \boldsymbol{\sigma}^2\mathbf{I})^{-1} \tag{21}$$

is the Kalman gain. Recalling (15) and defining

$$\begin{aligned}\mathbf{A}(n) &= \mathbf{F}\mathbf{D}^{1/2}\breve{\mathbf{R}}_g(n)\mathbf{D}^{1/2}\mathbf{F}^H = \\ &= \rho^2\mathbf{R}_e(n-1) + (1-\rho^2)\mathbf{R}_H,\end{aligned} \tag{22}$$

that coincides with (16b), we have from (19)

$$\begin{aligned}\mathbf{R}_e(n) &= \mathbf{A}(n) - \rho\mathbf{F}\mathbf{D}^{1/2}\mathbf{G}(n)\tilde{\mathbf{C}}(n)\breve{\mathbf{R}}_g(n)\mathbf{D}^{1/2}\mathbf{F}^H = \\ &= \mathbf{A}(n) - \rho\mathbf{F}\mathbf{D}^{1/2}\mathbf{G}(n)\tilde{\mathbf{X}}(n)\mathbf{A}(n),\end{aligned} \tag{23}$$

for the second equality we have used the definition $\tilde{\mathbf{C}}(n) = \tilde{\mathbf{X}}(n)\mathbf{F}\mathbf{D}^{1/2}$. Substituting (21) in (23) we finally get (16a).

VIII. Appendix-B

From (19), (20) and (21), the error correlation matrix can be written as (we drop the argument n for brevity)

$$\mathbf{R}_g = \breve{\mathbf{R}}_g - \rho^2\breve{\mathbf{R}}_g\tilde{\mathbf{C}}^H(\tilde{\mathbf{C}}\breve{\mathbf{R}}_g\tilde{\mathbf{C}}^H + \boldsymbol{\sigma}^2\mathbf{I})^{-1}\tilde{\mathbf{C}}\breve{\mathbf{R}}_g. \tag{24}$$

Applying the matrix inversion lemma, we get

$$\begin{aligned}\mathbf{R}_g &= \breve{\mathbf{R}}_g - \frac{\rho^2}{\sigma^2}[\breve{\mathbf{R}}_g\tilde{\mathbf{C}}^H\tilde{\mathbf{C}}\breve{\mathbf{R}}_g - \\ &\quad -\breve{\mathbf{R}}_g\tilde{\mathbf{C}}^H\tilde{\mathbf{C}}(\boldsymbol{\sigma}^2\breve{\mathbf{R}}_g^{-1} + \tilde{\mathbf{C}}^H\tilde{\mathbf{C}})^{-1}\tilde{\mathbf{C}}^H\tilde{\mathbf{C}}\breve{\mathbf{R}}_g)],\end{aligned} \tag{25}$$

that does not depend directly on $\tilde{\mathbf{X}}$ but only on the power allocation matrix $\mathbf{D}_x$, since $\tilde{\mathbf{C}}^H\tilde{\mathbf{C}} = \mathbf{D}^{1/2}\mathbf{F}^H\mathbf{D}_x\mathbf{F}\mathbf{D}^{1/2}$. Since $\mathbf{R}_g(n)$ and $\mathbf{R}_e(n)$ are related through (15), the same applies to error correlation matrix $\mathbf{R}_e(n)$.

References

[1] A. J. Goldsmith and P. Varaiya, ”Capacity of fading channels with channel side information,” *IEEE Trans. Inform. Theory*, vol. 43, pp. 1986-1992, Nov. 1997.

[2] T. Keller and L. Hanzo, ”Adaptive multicarrier modulation: a convenient framework for time-frequency processing in wireless communications,” *IEEE Proc. of the IEEE*, vol. 88, pp. 611-640, May 2000.

[3] A J. Goldsmith and S.-G. Chua, ”Variable-rate, variable-power MQAM for fading channels,” *IEEE Trans. Commun.*, vol. 45, pp. 1218-1230, Oct. 1997.

[4] S. Ye, R. S. Blum, L. J. Cimini Jr., ”Adaptive modulation for variable-rate OFDM systems with imperfect channel state information,” *Proc. VTC 2002,* pp. 768-771.

[5] ETSI TS 101 475, ”Broadband Radio Access Networks (BRAN); HIPERLAN Type 2; Physical (PHY) layer,” ETSI, Dec. 2001.

[6] IEEE Std 802.16c-2002 , ”IEEE Standard for Local and metropolitan area networks - Part 16: Air Interface for Fixed Broadband Wireless Access Systems-Amendment 1: Detailed System Profiles for 10-66 GHz”, 2002.

[7] I. Barhumi, G. Leus and M. Moonen, ”Optimal training design for MIMO OFDM systems in mobile wireless channels,” *IEEE Trans. on Signal Processing*, vol. 51, pp. 1615-1624, June 2003.

[8] A. Scaglione, G.B. Giannakis and S. Barbarossa, ”Redundant filterbank precoders and equalizers. I. Unification and optimal designs,” *IEEE Trans. on Signal Processing*, vol. 47, pp. 1988-2006, July 1999.

[9] B. Lu, X. Wang and K. R. Narayanan, “LDPC-based space-time coded OFDM systems over correlated fading channels: performance analysis and receiver design,” *IEEE Trans. Comm.*, vol. 50, no.1, pp. 74-88, Jan. 2002.

[10] B. Hassibi and B. M. Hochwald, “How Much Training is Needed in Multiple-Antenna Wireless Links?,” Technical Memorandum, Bell Laboratories, Lucent Technologies, April 2000 [available on line: mars.bell-labs.com/cm/ms/what/mars/papers/training/].

[11] S. Haykin, *Adaptive filter theory*, Prentice Hall, fourth edition, 2002.

[12] C. Komninakis, C. Fragouli, A. H. Sayed and R. D. Wesel, “Multi-input multi-output fading channel tracking and equalization using Kalman estimation,” *IEEE Trans. Signal Processing,* vol. 50, pp. 1065-1076, May 2002.